technische universität
dortmund

fakultät
physik

Effective Theory for Dark Matter Direct Detection at Dimension Seven

Master's thesis by

Aaron Gootjes-Dreesbach

born in Hagen

submitted to Lehrstuhl für Theoretische Physik IV
Fakultät Physik
TU Dortmund

supervised by Jun.-Prof. Dr. Joachim Brod

second corrector Prof. Dr. Heinrich Päs

December 2016

Abstract

An effective field theory approach is highly attractive for the model-independent treatment of dark matter searches and allows for the consistent inclusion of renormalization group running effects. We present a full basis for fermionic electroweak dark matter interactions at mass dimension seven as an extension of an EFT framework for direct detection searches by Bishara, Brod, Grinstein and Zupan. We also provide matching between the full basis and effective theories below the electroweak scale for both light and electroweak-scale dark matter.

Contents

1 Introduction

To discover the nature of dark matter and unveil what we can learn from it about physics beyond the standard model, we need to interpret and combine a spectrum of observables from highly diverse experiments. In direct detection searches, we also face the problem of the wide range of energy scales that are involved. This motivates the use of a tower of effective field theories for a model-independent phenomenological treatment of particle dark matter models. Bishara, Brod, Grinstein and Zupan [1, 2] developed such a framework to consistently incorporate leading order effects of dimension five and six UV operators with a fermionic, electroweak multiplet dark matter field.

In this thesis we seek to start extending this framework to effective operators of mass dimension six by providing a full basis of UV operators and matching this onto effective theories below the electroweak scale.

The remainder of this thesis is divided into eight chapters. The following two chapters are very brief reviews of effective field theory and dark matter, respectively, and serve to put this work into a broader context and introduce our notations and conventions. In chapter four, we present the effective field theory framework for direct detection searches that we will employ. In the fifth and sixth chapter, we discuss the formulation of a complete basis of dimension six operators in the UV and the subsequent matching of this basis to theories below the electroweak scale, for dark matter that is either very light or at the electroweak scale itself. We follow this with a discussion of methods we will use in the calculation of the anomalous dimension matrices for the UV renormalization group running of our operators. Finally, we discuss four different example UV models in chapter eight, which motivate the use of this work and give concluding remarks in chapter nine.

2 Review of Effective Field Theory

The development of effective field theories (EFTs) and the renormalization group is one of the most influential developments in physics in the second half of the last century. In essence, they are the systematic quantum field theory application of the general principle to describe physical systems using only the appropriate degrees of freedom and the most important phenomena. Important effects are commonly identified by finding comparable parameters of which some are very small compared to others. In QFT, the only available dimensioned parameters to compare are energies, or equivalently distances. It is natural, then, to attempt an *effective* description of a full theory by removing or simplifying high-energy, short-distance degrees of freedom beyond a mass scale M. Alternatively, dimensionless quantities like coupling strengths or velocities can be used to guide the construction of the effective theory.

To motivate the usefulness of EFTs we consider an n-loop amplitude of a process at an energy scale E and small coupling α. The loop integrals can give logarithmic contributions up to order $(\alpha \ln \frac{E}{\mu})^n$ when defining the renormalized couplings at a scale μ in a MS-like renormalization scheme. If the process happens at a scale E sufficiently removed from μ, these large logarithms clearly lead to the breakdown of perturbation theory. In EFTs, we will see that we can connect different scales in a renormalization-group improved perturbation theory, effectively summing the contributions from the leading logarithms to all orders.

In the first section in this chapter, we discuss the two different major use cases of EFT, building theories from the top down or bottom up. The second section presents the Wilsonian and Continuum paradigms in the explicit construction of low-energy effective theories. The last section introduces the beta function and anomalous dimensions and gives details on renormalization group improved perturbation theory.

For a more in-depth discussion of effective field theory, consider [3], [4] and Georgi's review [5].

2.1 Top-down vs bottom-up approach

In a *top-down* approach to EFTs, we start by specifying the full theory at high energies including all of its parameters. We then use one or more effective theories valid at lower energy scales to benefit from their greater computational potential or convenience, postponing for now how to find appropriate effective descriptions. Whenever we switch from a full to an effective theory, we need to make sure that they agree at the intersection of their respective ranges of applicability. The natural criterion for quantum field theories is to demand that their physical S-matrix elements at this energy scale are equal within the precision of our calculation, which usually allows us to fix the parameters of the low-energy theory using those at high energies in a process called matching.

Taking QFT seriously as a physical model up to high energies allows us to take an alternative *bottom-up* approach to EFTs. We recognize that the best description we have for the shortest distances is likely to be just an effective theory approximating a more fundamental description at low energies. Taking this view makes it unnecessary to demand renormalizability in the traditional sense, since we do not expect to find a theory valid at all energies. As we will see later, removing heavy degrees of freedom naturally leads to new terms in the Lagrangian that are not renormalizable in the strict sense, suppressed with powers of the high scale. Note that this does not jeopardize the predictiveness of the EFT because we seek to only approximate the full theory up to a fixed order in the small parameters. The great advantage of this approach is the possibility to construct a predictive framework from only very general assumptions about the unknown theory at higher scales: Starting from the field content and symmetries one expects to hold, as well as a scheme to distinguish operators by the size of their effects on dynamics, we can extend the Lagrangian of our current theory with full sets of increasingly negligible operators:

$$\mathcal{L}_0 \to \mathcal{L}_0 + \mathcal{L}_{eff}^{(1)} + \mathcal{L}_{eff}^{(2)} + \dots$$

Since operators of a higher mass than spacetime dimension must come with inverse powers of a mass scale, usually relevant to the short-distance physics, this is a natural power-counting scheme to consider. Such a framework now allows one to bring together different experimental results and interpret them in a largely model-independent way.

In this thesis and future work, we employ both approaches at different stages in our analysis: To capture the effects of a broad class of dark matter models, we construct an effective extension of the Standard Model from the bottom-up. In order to then derive predictions for experiments performed at very low energies after crossing energies associated with wildly different physics, we require a tower of effective theories for each respective scale.

2.2 Wilsonian vs Continuum EFT

Kenneth Wilson's ansatz to construct an effective theory from a full theory of fields ϕ governed by the action $S[\phi]$ is to split the fields into short-distance modes $\phi_>$ and long-distance modes $\phi_<$ with respect to a cut-off energy scale Λ, and perform the functional integral over the heavy degrees of freedom to determine an effective action $S_\Lambda[\phi_<]$ for the remaining long-distance physics:

$$\int \mathcal{D}\phi_< \int \mathcal{D}\phi_> \, e^{\mathrm{i}S[\phi_< + \phi_>]} \stackrel{!}{=} \int \mathcal{D}\phi_< \, e^{\mathrm{i}S_\Lambda[\phi_<]}. \tag{1}$$

This averaging procedure in general generates non-local interactions between the remaining light degrees of freedom, which can be expressed as a series of non-renormalizable local interactions in an operator product expansion (OPE). The respective coefficients are called *Wilson coefficients*. Additionally, this expansion can give new contributions to the existing parameters of the theory.

This construction naturally leads to a cut-off regularization at the scale Λ, which in turn requires a mass-dependent subtraction scheme.

One of the advantages to this philosophy is not only its intuitive ansatz, but that the *Appelquist-Carazzone decoupling theorem* [6] guarantees that there is a physical renormalization scheme for simple theories like QED that leads to the decoupling of heavy degrees of freedom at scales sufficiently below their mass. The only effect of their existence, up to terms that are suppressed by the ratio of the energies involved in a given process and their mass, is the modification of the theory's parameters. This is not trivial, since integration over loop momenta could in principle probe all scales, and affirms the philosophy that it is possible to build theories without taking into account all microscopic fluctuations.

This decoupling can be understood with the beta functions and anomalous dimensions describing the evolution of the theory's parameters along energy scales. These explicitly depend on the mass scales of all particles, allowing appropriate couplings to tend to zero after passing a particle's mass. Taking the point of view of the path integral, this corresponds to *all* modes of a massive particle being averaged over in the construction of the effective action due to the mass gap.

However, computationally, there are significant drawbacks: Cut-off regularization clearly breaks the manifest Poincare and gauge invariance and simple power-counting breaks down, complicating loop calculations.

Instead, one would often like to work in the manifestly covariant dimensional regularization with a mass-independent renormalization scheme, which the *Continuum EFT* approach allows. Here, the effective theory starts out as just the dimensionally regulated full theory where the artificial mass scale μ is chosen to equal the scale E which we want to describe. This is an appropriate choice, because the problematic large logarithms discussed in the introduction to this chapter vanish. Additionally, while this setup does not eliminate any degrees of freedom like a cut-off at E would, it does modify the behaviour of the theory in the same short-distance regime beyond E [5]. Now, evolving the parameters down does not correspond to integrating out more and more degrees of freedom, but instead corresponds to modifying the dynamics already at lower and lower energies, all the while preserving Poincare invariance.

This procedure runs into a problem as soon as we pass the threshold of a particle mass: Using a mass-independent renormalization scheme like $\overline{MS}$ prevents the automatic decoupling of heavy degrees of freedom through the evolution functions, which are discussed in more detail in the next section. Instead, the threshold mass dependence has to be put in by hand: Whenever we pass a particle mass threshold, we remove that particle from our theory. Using the same procedure as in Wilsonian EFT, but for the whole particle at once, now induces new effective operators between the remaining fields, which we can determine in exactly the same matching procedure.

While both approaches can be considered ultimately equivalent, we follow the continuum method in this thesis due to its calculational convenience.

2.3 Beta functions and anomalous dimensions

We now want to discuss in detail how to perform the evolution down energy scales between masses. Here, we consider a mass-independent scheme in dimensional regularization, but the procedure works very analogously in Wilsonian EFTs.

Given a coupling constant and mass renormalized by $g_0 = Z_g\, g\, \mu^\epsilon$ and $m_0 = Z_m m$, we start from the fact that the bare coupling is independent of μ. This leads us immediately to the renormalization group equations in $d = 4 - 2\epsilon$-dimensional regularization as

$$\frac{\mathrm{d}g(\mu)}{\mathrm{d}\ln\mu} = \beta(g(\mu)) - \epsilon g(\mu), \qquad \frac{\mathrm{d}m(\mu)}{\mathrm{d}\ln\mu} = -\gamma_m(g(\mu))\, m(\mu), \tag{2}$$

where

$$\beta(g) = -g\frac{1}{Z_g}\frac{\mathrm{d}Z_g}{\mathrm{d}\ln\mu}, \qquad \gamma_m(g) = \frac{1}{Z_m}\frac{\mathrm{d}Z_m}{\mathrm{d}\ln\mu} \tag{3}$$

are the beta function and anomalous dimension of the mass. In our mass-independent renormalization scheme we can derive these function directly from the $1/\epsilon$ pole of the renormalization constants:

$$\beta(g) = 2g^3\frac{\mathrm{d}Z_g^{(1)}(g)}{\mathrm{d}g^2}, \qquad \gamma_m(g) = -2g^2\frac{\mathrm{d}Z_m^{(1)}(g)}{\mathrm{d}g^2}, \tag{4}$$

where the superscript (1) indicates the terms proportional to $1/\epsilon$. For the detailed derivation of these formulae, see e.g. Buras [4]. The RG equations can alternatively, from an EFT point of view, be seen to be a result of 'continuously' matching a theory at μ down to a theory at $\mu - \mathrm{d}\mu$.

Solving these evolution equations allows us to compare coupling constants and masses defined at different scales as well as experimental observations performed for different energies, without introducing the large logarithms that we discussed in the introduction to this chapter. A detailed analysis shows that even though the beta function is calculated to a fixed order, this approach effectively includes the leading logarithmic corrections to all orders.

In an EFT, we often additionally have to renormalize the Wilson coefficients and calculate their running. We write the unrenormalized Wilson coefficients as a row vector C_0 and the respective operators as a column vector Q_0, so that the non-renormalizable part of the Lagrangian can be written as $C_0 Q_0$. To renormalize, we introduce the matrix Z, since one operator can in general generate contributions to others at loop level:

$$C_0 = CZ. \tag{5}$$

Note that in the calculation of the renormalization matrix, the contributions of replacing the unrenormalized fields and couplings in Q_0 must be taken into account. We can again derive renormalization group equations from the scale-independence of the bare coefficients:

$$\frac{\mathrm{d}C(\mu)}{\mathrm{d}\ln\mu} = \gamma^T(g_i(\mu))\, C(\mu), \tag{6}$$

where the anomalous dimension matrix γ is given by

$$\gamma = Z\,\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\,Z^{-1}. \tag{7}$$

Working in the $\overline{MS}$ scheme at lowest order in the couplings and expanding the renormalization matrix in ϵ poles, we find that at one-loop

$$\gamma = 2Z^{(1)}, \tag{8}$$

where $Z^{(1)}$ is the part of Z that scales with $1/\epsilon$.

Note that the necessity of allowing non-diagonal elements opens the possibility of *operator mixing*: During the running, operators with large matrix elements can mix into operators with small ones, non-trivially affecting the phenomenology of the theory at low scales.

3 Review of Dark Matter

While evidence for the existence of dark matter is numerous, the exact nature of this phenomenon remains elusive and emerged as one of the central problems of modern fundamental physics. It is one of the clearest signs motivating the necessity of physics beyond the standard model and solving this puzzle is likely to open novel research venues. The umbrella term 'dark matter' is applied to forms of matter that do not significantly interact electromagnetically, but whose gravitational influence accounts for a plethora of experimental observations.

The remainder of this short review chapter is divided into three sections: First, we go through some of the indirect experimental observations suggesting dark matter. Second, we give a very rough overview of different theories incorporating it. And last, we discuss the strategies and results of active searches for dark matter.

3.1 Experimental status

Useful reviews that provide much more detail on existing experimental research can be found, for instance, in [7] or [8].

3.1.1 Direct gravitational anomalies

The first experimental indication of dark matter that essentially held true were Fritz Zwicky's 1933 observations of the Coma galaxy cluster [9]. Using the virial theorem to estimate its total mass from red-shift velocity measurements, he concluded that the significantly smaller mass of visible matter would have to be supplemented by non-luminous matter.

Since then, this line of evidence has been reinforced through the study of many more clusters. Moreover, cluster masses and thereby the ratio of luminous to dark matter is measured in two additional, independent ways: On one hand, space probes like the Chandra X-ray observatory allow for the estimation of densities of hot gases in galaxy clusters from their thermal X-ray radiation, which can be used to infer the cluster mass since the thermal pressure must be counterbalanced by gravity. On the other hand, gravitational lensing can be used for this purpose - as per general relativity, the clusters' total mass distorts the light coming from sources behind it.

A particularly convincing example is the study of the Bullet Cluster, which is formed from two clusters that, on a cosmological timescale, collided recently. This led to a separation of the stars' and gases' centers of mass, since only the gases palpably interacted with each other. The latter has the higher portion of the luminous mass, but gravitational lensing shows mass peaks close to the stars. Taking into account also a massive halo of negligibly interacting dark matter would predict exactly this, while theories that postulate modifications of Newtonian or Einsteinian dynamics struggle to explain this.

Rubin and Ford [10] introduced another strong argument for dark matter from the motion *within* galaxies while studying galaxy rotation curves in the 1960s. These plot the mean orbital speeds of stars and hydrogen gas against the distance from the galactic core, measured using red-shift spectroscopy in edge-on galaxies. Assuming a Keplerian orbit, one would expect a scaling behaviour of $v \sim 1/\sqrt{r}$ since most of the mass is located near the galactic core, but measurements instead show an approximately constant velocity of $v \approx 240$ km/s [7]. This is consistent with a halo of dark matter surrounding the luminous matter, with a density scaling like $\rho \sim 1/r^2$ in its vicinity. Using lensing, the existence of a dark matter halo can be inferred on larger distances from the galactic core.

The density of dark matter in our local neighbourhood is especially important in the context of direct detection measurements. This can be estimated by tracking the movement of stars in the vicinity of the sun or on a larger scale using rotation curves of the Milky Way. The first approach suffers from large statistical uncertainties, while the second approach is somewhat challenging from within the Milky Way, and heavily depends on assumptions on the shape of its dark matter halo[1]. A review [11] lists recent determinations, which usually fall between 0.2 and 0.5 GeV cm^{-3}. The local velocity distribution of dark matter can be estimated with simulations, yielding an approximately Maxwellian distribution, differing mostly through a more pronounced tail in high velocities. This could especially affect light dark matter measurements. The distribution is usually described by its average velocity of roughly 270 km/s [8].

3.1.2 Cosmological arguments

Beyond observations of current gravitational dynamics on large scales, the influence of dark matter on the evolution of the universe opens the possibility of finding effects in cosmological observables. Competing theories are usually distinguished in a simultaneous fit of a large number of such observables. The best fit supports the so-called ΛCDM model, which posits the existence of a cosmological constant Λ together with cold, i.e. non-relativistic, dark matter. The Particle Data Group [7] finds the following values for the relic mass densities Ω, normalized to the critical density of a flat universe:

$$\Omega_{\mathrm{nbm}} h^2 = 0.1168 \pm 0.0020, \qquad \Omega_{\mathrm{b}} h^2 = 0.02226 \pm 0.0020, \tag{9}$$

where the left and right values represent non-baryonic dark matter and baryonic matter, respectively, and $h = \frac{H}{100} \frac{\mathrm{s \cdot Mpc}}{\mathrm{km}}$ with the Hubble constant H. While the term 'dark matter' can encompass different phenomena, this suggests that at most a small fraction of dark matter will consist of non-luminous baryonic matter.

The rest of this subsection will discuss how some specific observations have an effect on this best fit and thereby underpin the existence of dark matter.

The Cosmic Microwave Background (CMB), which was predicted by Gamow in the 1940s and first detected by Penzias and Wilson in 1964, consists of thermal photons from the early universe that propagate unimpeded since recombination decoupled

[1] Combining these different measures can conversely yield information about the dark matter halo distribution, usually favoring a spherical halo [11].

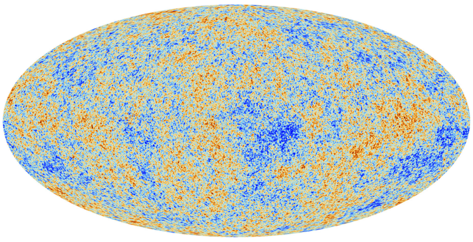
them from matter. While originally peaking in visible and UV radiation, the expansion of the universe caused a redshift to the microwave spectrum with an equivalent temperature of 2.7 K. It is remarkably isotropic, but its minuscule angular anisotropies of the order of 10^{-5} allow us to probe the physics of the early universe. This is usually studied by expanding the fluctuations in spherical harmonics, resulting in a multipole spectrum, which can be predicted in specific models of cosmic evolution. One example relevant to dark matter physics are the *acoustic oscillation* peaks in this spectrum, roughly between the 100th and 1000th multipole moments. The inhomogeneities led to restoring oscillations in the proton-electron plasma while still coupled to the photons, which we observe after they freeze out. Since the multipoles are associated with different length scales, and these oscillations take longer the larger the scale we are considering is, higher multipoles correspond to measuring the anisotropy after a longer timespan of oscillations. The shape of the spectrum is thereby directly connected to the dynamics of an oscillating plasma over time. Every peak alternatingly represents either the point in time when the collapsing plasma led to an overdense region or when the expanding plasma led to an underdense region. The presence of dark matter, which does not build internal pressure and therefore clumps easier, leads to a more pronounced anisotropy for those (odd-numbered) peaks, where the dark matter supports the oscillatory collapse with its gravity.

Today's best measurements of the CMB spectrum stem from satellite probes, namely the COBE, WMAP and most recently the Planck [13] experiments, whose results are depicted in Fig. 1.

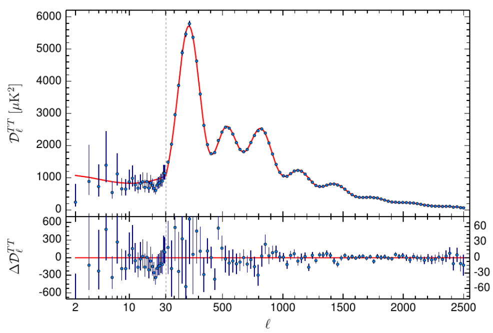
The relative abundances of elements resulting from big bang nucleosynthesis depend only on the baryon-to-photon ratio, which makes it highly sensitive to the density of baryonic matter at the point of its inset. This allows us to calculate constraints on the baryonic matter density of roughly [14]

$$0.018 < \Omega_{\mathrm{b}} h^2 < 0.023. \tag{10}$$

Note that this assumes the baryonic dark matter to be available to this process, which is e.g. not the case for primordial black holes.



(a) Temperature fluctuations across the sky



(b) Angular power spectrum

FIG. 1. Planck measurements of the Cosmic Microwave Background [12].

Furthermore, the existence of dark matter heavily affects the formation of large-scale structures out of the small inhomogeneities in the early universe, which can be simulated numerically or semi-analytically. The propensity of dark matter to clump allows it to form a backbone on which normal matter can accumulate faster than without it. These simulations typically suggest cold dark matter, which extends as filaments beyond the halos of galaxies and clusters.

Distance and acceleration measurements with Type Ia supernovae strongly constrain the dark energy and thereby indirectly also the dark matter relic density.

3.2 Dark matter models

The most active avenue of research in building theories of dark matter is the *particle dark matter* paradigm, which seeks to extend the standard model with additional fields whose excitations make up the observed dark matter halo. We give a few examples of such models:

WIMPs are *weakly interacting massive particles.* Their popularity is motivated by the so-called 'WIMP miracle', which refers to the observation that the relic abundance of dark matter observed today could be naturally explained by a particle with a mass of hundreds of GeV that couples to the standard model with a force of roughly the strength of the weak force and freezes out in the early universe. Moreover, supersymmetric extensions of the standard model readily provide such particles, such as neutralinos, gravitinos, gauginos or admixtures, whose stability can be naturally ensured by R-parity as the *lightest supersymmetric particle.* Similarly, models with extra dimensions can supply WIMPs as *lightest Kaluza-Klein particle.*

Axions are Goldstone bosons related to an additional, spontaneously broken $U(1)$ symmetry. This symmetry was proposed in 1977 by Peccei and Quinn [15] to explain the strong CP problem in QCD, i.e. the very small value of the CP-violating θ term. While the original model was ruled out, similar mechanics remain viable candidates to give a simultaneous explanation of cold dark matter. Many Axion-specific searches such as the ALPS experiment [16] try to detect Axions through the possibility of turning photons into axions and vice-versa in the presence of very strong magnetic fields.

Neutrinos have been proposed as dark matter candidates, especially in the context of the standard seesaw mechanism, which would explain small neutrino masses but also posit them to be Majorana particles and imply the existence of right-handed neutrinos. These would be *sterile*, i.e. only interacting gravitationally. However, as an example of a hot dark matter theory, they could only make up a fraction of the total dark matter, due to the cosmological considerations mentioned in the last section.

We also want to give two examples outside of the particle dark matter paradigm:

MACHOs are *massive compact halo objects* such as black holes, neutron or very faint stars. Searches for such objects in the Milky Way using gravitational lensing exclude a large range of reasonable models. Usually, MACHOs are counted as baryonic dark matter, although primordial black holes, which did not affect nucleosynthesis, are often categorized as non-baryonic.

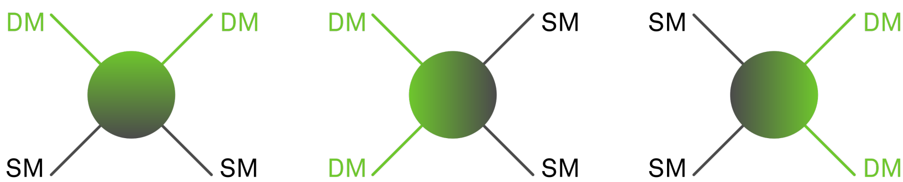
Topological defects such as cosmic strings can arise as remnants of phase transitions in the early universe, and can carry very significant masses. Possible experimental signatures include the distinct lensing they would produce as well as the 'loops' they can radiate through oscillations and collisions, which would decay to gravitational waves that could in turn be detected in gravitational observatories. While they are already predicted to rarely occur from our current understanding of the universe's evolution, the Gaussian nature of the CMB anisotropies strongly suggests that they make up at most a small part of the anomalies associated with dark matter.

An entirely different approach to the experimental findings is given by **Modified dynamics theories**, which posit that the relativistic or approximated Newtonian treatment used to interpret the observations of the last section has to be corrected in an encompassing theory. The most prominent theory was Milgrom's Modified Newtonian Dynamics (MOND, [17]), which modifies Newton's laws for extremely small accelerations for which it is not well-tested. However, it suffered from being inherently non-relativistic and in predicting the behaviour of clusters and the CMB. Another very recent attempt is Verlinde's emergent quantum gravity [18].

A challenge to these theories is usually to give equally good predictions as dark matter models to situations where the different behaviour of luminous and dark matter leads to a separation between them. An example would be the Bullet Cluster discussed in the last section, as well as the difference between odd- and even-numbered oscillations in the CMB.

3.3 Searches for dark matter

The different approaches to general searches for dark matter particles can be roughly categorized as either (i) direct detection experiments, where we look for scattering between the invisible, relic dark matter and standard model particles, (ii) indirect detection, where we look for annihilation products of accumulated dark matter, or (iii) collider searches, where the production of dark matter particles is attempted in high energy collisions. This is visualised in Fig. 2 and will be discussed one by one in the following three subsections.



(a) Direct detection (b) Indirect detection (c) Collider searches

FIG. 2. Visualisation of dark matter search paradigms. Time always flows to the right.

3.3.1 Indirect detection

The indirect detection approach argues that dark matter particles accumulate in the gravitational potential of heavy objects such as the galactic core or stars. This higher than average density in many specific models allows for an appreciable increase of the annihilation rate into standard model final states. Additionally, one could expect to find decay products for dark matter models that predict decays with a sufficiently long lifetime.

The (possibly secondary) products usually are stable particles like electrons, positrons, photons, neutrinos or (anti-)protons; antimatter in particular having the advantage of lower backgrounds. Experimentally, they are picked up by detectors for gamma rays, neutrinos or cosmic rays. This includes cases where the final state particles interact with other material themselves, such as positrons annihilating with the interstellar plasma and producing a typical 511 eV signal.

Gamma rays have the advantage of a clear spatial origin and preserving spectral information. However, the earth's opacity in these frequencies makes it necessary to perform observations either directly from space, a current example being the Fermi Gamma-ray Space Telescope[19], or using ground-based atmospheric Cherenkov telescopes.

Neutrino detectors obviously require massive amounts of detector material to accumulate sufficient data by collecting Cherenkov photons that result from products of neutrino interactions. They share the advantages of gamma ray searches. A currently competitive experiment is the IceCube neutrino observatory [20].

A challenge to both these types of searches is the uncertainty associated with the expected background signals from unrelated astrophysical processes in the origin regions, which are often imperfectly understood by themselves. This poses less of a problem for cosmic ray experiments, which exhibit a much smaller background especially for antimatter. But they do not provide spatial information on the source of the signal due to the complex dynamics of cosmic ray propagation. An example of a detector is the Alpha Magnetic Spectrometer mounted on the International Space Station [21].

A sufficiently strong signal in any of the spatially resolved experiments would have the additional advantage that they allow for independent probes into the dark matter distribution. A more thorough discussion of current indirect detection experiments can be found in [22].

3.3.2 Collider searches

If the center of mass energy is sufficiently high, we expect to pair produce dark matter particles in collider experiments. The central indication for their production would be signatures with missing energy or momentum, since the products would be invisible to the detectors and stable on cosmological lifetimes. This is often implemented in monojet searches, because they allow to select events without any other visible particles in the final state and the jet recoils visibly against the missing momentum. The jet is easily attached via a gluon radiating of the initial state quarks without interfering with the dark matter production itself.

A drawback of this approach is that it is impossible to determine from only this signal whether the new particle really contributes sizably to the dark matter responsible for the experimentally observed anomalies. That is, one can only find dark matter *candidates*. Collider experiments are also complementary to other searches in that they are better suited to detect light dark matter scenarios and limits can be combined in effective field theory approaches or simplified models.

3.3.3 Direct detection

Direct detection experiments aim to observe the recoil in normal matter resulting from a collision with a dark matter particle within a detector. Due to the small cross-section, experiments aim to comprise a very large volume of detector material. To keep the background from radiation and especially cosmic rays minimal, they are located in underground facilities.

The central observable of these experiments is the number and deposited energy of nuclear recoil events. Disregarding background events, this connects to the theory through the differential recoil rate

$$\frac{\mathrm{d}R}{\mathrm{d}E} = \frac{\rho}{m_A m_\chi} \int_{v_{\min}}^{v_{esc}} \mathrm{d}^3v \; v \; f(v) \; \frac{\mathrm{d}\sigma}{\mathrm{d}E}(v, E), \tag{11}$$

where ρ is the dark matter density in our immediate vicinity, m_χ is its mass, $f(v)$ its velocity distribution and σ the interaction cross-section with the given nucleus of mass m_A [23]. The integral runs from the smallest kinematically allowed velocity $v_{\min}$ to the galactic escape velocity v_{esc}. In order to interpret nuclear recoil data, we clearly need independent information on the dark matter density and distribution from the sources discussed earlier in this chapter.

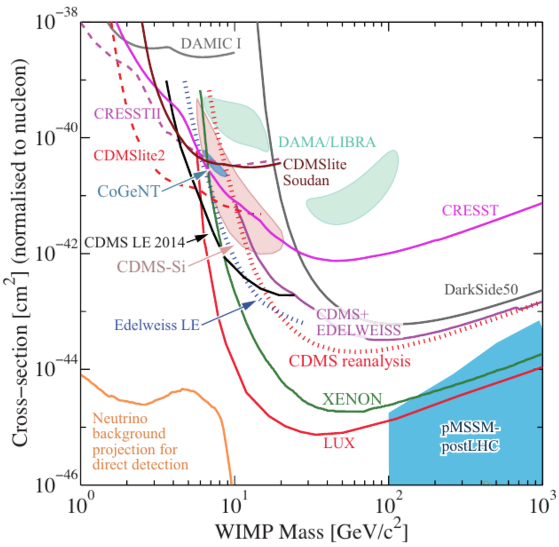


FIG. 3. PDG compilation of WIMP direct detection exclusion limits [7]. The cross section is given for spin-independent coupling. The deep blue region is an ATLAS scan of the MSSM parameter space, the other shaded regions are areas of supposed signal events.

In direct detection experiments, the energy of an event is essentially deposited into one of three channels: Photons from scintillation, electrons from ionization or vibrations. Many experiments access two of these channels to distinguish between electronic and nuclear recoil signals, and fall into roughly two categories:

Experiments like EDELWEISS or CRESST use cryogenically cooled crystals to detect phonons created by the impact. The former additionally measures the number of electrons produced by ionization, while the latter uses scintillating crystals to detect photons.

Noble liquid detectors, like the LUX experiment, detect both the scintillation resulting from the impact as well as a time-delayed proportional scintillation signal from ionization electrons.

The current exclusion limits from direct detection experiments as collected by the PDG are displayed in Fig. 3.

Since the experiments are usually sensitive to recoil energies in the $\mathcal{O}(10\ \mathrm{keV})$ range and the nucleon mass is known, the momentum transfer is bounded from above at roughly 200 MeV [1].

4 An EFT framework for direct detection searches

Vastly different scales of energy are involved in a typical direct detection experiment. For a consistent treatment of the detection phenomenology in a systematic framework, a tower of effective field theories is therefore a very appealing setup, connecting all scales from the theory at high energies, down to the nuclear scale involved in the detection itself. We align our treatment with such a framework described by Bishara, Brod, Grinstein and Zupan [1, 2], extending its scope to effective operators of mass dimension seven in the UV.

The different theories in this description are displayed in Fig. 4. Interactions between DM and SM are either weak gauge interactions or non-renormalizable effective operators induced by a mediator sector at a heavy scale Λ. After passing the electroweak scale v_{EW}, we express the theory in the broken fields and integrate out the top quark and massive gauge bosons. We treat two cases, where the dark matter is either very light or comparable to v_{EW}. In the latter case, we also transition to Heavy Dark Matter Effective Theory (HDMET). Thereafter, we pass the bottom and charm quark mass thresholds, integrating out the respective particles along the way.

Once we arrive at the scale of chiral symmetry breaking $\Lambda_\chi \approx 1$ GeV, QCD perturbation theory starts to break down. To remedy this, we need to nonperturbatively match onto theories at lower energies. First, a (Heavy Baryon) Chiral Perturbation Theory (HBChPT) that describes the dark matter interactions with nucleons and pions needs to be constructed. This in turn is used, after extracting the most important contributions using chiral counting in a Chiral Effective Theory (ChEFT), to calculate the coefficients in an effective theory describing the DM-nucleus scattering. This procedure at low energies is analogous to the treatment in [2], but needs to be extended for our analysis, which we relegate to future work.

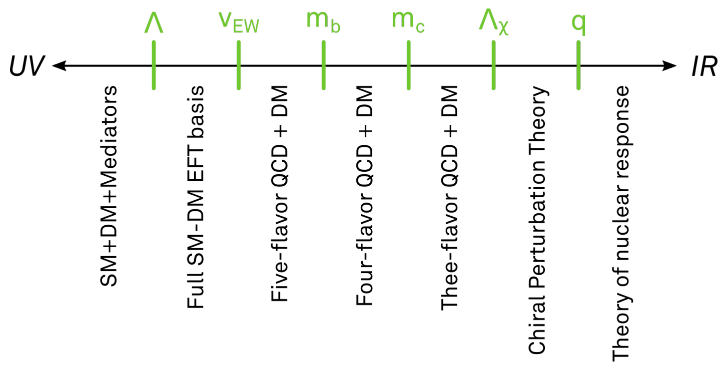


FIG. 4. Visualization of all scales and theories in the EFT framework, adapted from [1].

Taking into account the renormalization group flow of Wilson coefficients is phenomenologically important even for order-of-magnitude estimations: UV Wilson coefficients that would naïvely contribute only to velocity-suppressed interactions in the non-relativistic limit can mix into unsuppressed matrix elements during RG running. The same can also result from a coherence effect that leads to the enhancement of spin-independent matrix elements with respect to spin-dependent ones by the square of the nucleon number of the nucleus involved in direct detection experiments. The importance of these effects have been discussed elsewhere in more detail, e.g. [24, 25].

This framework exhibits the usual advantage of EFTs in that it is possible to combine observational limits from different experimental searches as constraints on the UV Wilson coefficients without limiting oneself to any particular model of dark matter. Of course, there are factors limiting the range of applicability of this approach: For direct detection experiments, the mediator scale must at least be higher than the momenta exchanges, i.e. ~ 200 MeV. For indirect detection, it must be larger than the dark matter mass scale, at which its annihilation or decay happens. For collider experiments, the mediator scale must exceed the collision energy, which for reasonable Λ prevents the use of this framework with LHC data.

In the first section of this chapter, we will go through the details of the dark matter scenario covered by our analysis. The remaining three sections describe the effective theories at very high energies, directly below the electroweak scale and at low energies, respectively.

4.1 Our dark matter scenario

Our effective description covers a single fermionic particle χ, either Dirac or Majorana, that transforms trivially under $SU(3)$ and according to an irreducible representation of the weak $SU(2)$. The candidate for dark matter would be given by the uncharged component χ^0 after electroweak symmetry breaking. Since direct detection experiments exclude additional Dirac particles coupling to the Z boson up to very high energies (see e.g. [26])[2], we conclude χ must have zero hypercharge. It then follows by the Gell-Mann Nishijima relation $Q = \tau^3 + Y_\chi$ that in order to be able to find an electrically neutral component of χ, it must transform under an odd-dimensional irreducible representation of $SU(2)$. We do not put constraints on m_χ by demanding it to be a thermal relic.

We furthermore assume the existence of a symmetry such as $\mathcal{Z}_2$ to explain the stability of the dark matter particles. This has the effect of disregarding effective operators with an odd number of χ particles.

To simplify our analysis, this work specifically assumes that the Wilson coefficients at dimension five and six in [1] all vanish, since any sizeable coefficient would likely mean that the treatment in [1] would suffice to search for this model using those operators.

We give a few example models that can be described in this framework in chapter 8.

[2]We do not consider the more complex scenario where the DM can arise as an admixture of different multiplets here, for which we cannot rule out $Y_\chi \neq 0$ this way.

	$SU(3)$	$SU(2)$	$U(1)$
Q	3	2	+1/6
L	1	2	-1/2
U	3	1	+2/3
D	3	1	-1/3
E	1	1	-1
G	8	1	0
W	1	3	0
B	1	1	0
H	1	2	+1/2
χ	1	$N_\chi^{SU(2)}$	Y_χ

TABLE I. The particle content of the effective theory in the UV. The Q and L fields are left-handed Weyl fermions, while the other fields in the first block are right handed. The fields in the second block are the gauge bosons corresponding to the SM symmetries, H is a spin 0 scalar and χ is the spin 1/2 dark matter field (either Majorana or Dirac). For $U(1)$, we give the hypercharge, while for the other gauge groups we give the dimension of the irreducible representation. We use four-component notation for all fermions. $N_\chi^{SU(2)} = 2J_\chi + 1$ is the dimension of the DM representation of $SU(2)$.

4.2 Effective description at high energies

The full particle content of our UV theory is given in Tab. I, including their representations under the gauge group $U(1) \times SU(2) \times SU(3)$.

For reference, the Lagrangian is given by [3]

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}^{\text{gauge}} + \mathcal{L}^{\text{higgs}} + \mathcal{L}^{\text{matter}} + \mathcal{L}^{\text{yukawa}} + \mathcal{L}^{\text{DM}} + \mathcal{L}^{\text{eff}} \\
\mathcal{L}^{\text{gauge}} &= -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}W_a^{\mu\nu}W^a_{\mu\nu} - \frac{1}{4}G_a^{\mu\nu}G^a_{\mu\nu} \\
&\qquad \text{where} \begin{cases} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu \\ G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_3 f^{abc} G^b_\mu G^c_\nu \end{cases} \\
\mathcal{L}^{\text{higgs}} &= |D_\mu H|^2 - V(H) \\
&\qquad \text{where } V(H) = -\mu^2 H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2 \\
\mathcal{L}^{\text{matter}} &= \sum_{k=1}^{N_g} \left(\bar{Q}_k \mathrm{i}\not{D} Q_k + \bar{L}_k \mathrm{i}\not{D} L_k + \bar{U}_k \mathrm{i}\not{D} U_k + \bar{D}_k \mathrm{i}\not{D} D_k + \bar{E}_k \mathrm{i}\not{D} E_k \right) \\
\mathcal{L}^{\text{yukawa}} &= -y_t \bar{Q}^a_3 \epsilon^{ab} H^{\dagger b} U_3 + \text{h.c.}
\end{aligned}
$$

[3] To set our conventions, the Clifford-Algebra of gamma matrices is given by $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$, where $\eta^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$. We define $\sigma^{\mu\nu} := \frac{\mathrm{i}}{2}[\gamma^\mu, \gamma^\nu], \gamma_5 := \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$, and choose the Levi-Civita tensor to fulfill $\varepsilon^{0123} = 1$. The left and right-handed projectors are given by $P_{R/L} = (1 \pm \gamma_5)/2$.

Greek indices are Lorentz indices while latin indices are either adjoint SU(2) or SU(3) indices, depending on the context, and ϵ^{abc} and f^{abc} are the respective structure constants. The covariant derivative is given by

$$D_\mu := \partial_\mu - \mathrm{i}g_1 B_\mu Y - \mathrm{i}g_2 W^a_\mu \tau^a - \mathrm{i}g_3 G^b_\mu t^b, \tag{12}$$

where Y is the hypercharge of the field it is acting on and τ^a and t^b are the generators in the appropriate representation of $SU(2)$ and $SU(3)$, respectively.

The dark matter Lagrangian $\mathcal{L}^{\mathrm{DM}}$ is given by

$$\mathcal{L}^{\mathrm{DM}} = \bar{\chi}(\mathrm{i}\not{D} - m_\chi)\chi \tag{13}$$

for the case of Dirac particles and

$$\mathcal{L}^{\mathrm{DM}} = \frac{1}{2}\,\bar{\chi}^c(\mathrm{i}\not{D} - m_\chi)\chi \tag{14}$$

for Majorana dark matter, which fulfills the Majorana condition $\chi^c = \chi$.

The effective Lagrangian $\mathcal{L}^{\mathrm{eff}}$ contains a full operator basis at the given mass dimension of interest and will be discussed in more detail in chapter 5, where we construct this basis.

Note that we keep our discussion minimal by requiring all interactions to be flavor-diagonal and only keep the top Yukawa coupling. Additional terms that appear in the Lagrangian through a gauge-fixing procedure will be discussed in chapter 7.

Classical equations of motion

For the sake of completeness, we now list the equations of motions derived from the Lagrangian using standard functional methods. For the fermionic fields of the standard model

$$\mathrm{i}\not{D}\, Q = 0, \qquad \mathrm{i}\not{D}\, L = 0, \tag{15}$$

$$\mathrm{i}\not{D}\, U_i = \delta_i^3 y_t \epsilon H^\dagger U_3, \qquad \mathrm{i}\not{D}\, D = 0, \tag{16}$$

$$\mathrm{i}\not{D}\, E = 0 \tag{17}$$

hold, as well as

$$D_\mu D^\mu\, H = \left(\mu^2 - \frac{\lambda}{2}\, H^\dagger H\right) H \tag{18}$$

for the Higgs boson, while the gauge field strength tensors fulfill

$$D^\mu B_{\mu\nu} = \partial^\mu B_{\mu\nu} = -g_1 \sum_f Y_f \bar{f}\gamma_\nu f + \mathrm{i}\frac{g_1}{2} H^\dagger \overleftrightarrow{D}_\nu\, H, \tag{19}$$

$$D^\mu W^a_{\mu\nu} = \left(\partial^\mu \delta^{ab} - g_2 \epsilon^{abc} W^{\mu,c}\right) W^b_{\mu\nu} = -g_2 \sum_f \bar{f}\tau^a\gamma_\nu f + \mathrm{i}g_2 H^\dagger \tau^a \overleftrightarrow{D}_\nu\, H, \tag{20}$$

$$D^\mu G^a_{\mu\nu} = \left(\partial^\mu \delta^{ab} - g_3 f^{abc} G^{\mu,c}\right) G^b_{\mu\nu} = -g_3 \sum_f \bar{f} t^a \gamma_\nu f, \tag{21}$$

where f is any fermion and $\overleftrightarrow{D}_\mu := \overleftarrow{D}_\mu - D_\mu$. For a Dirac or Majorana dark matter field, the Dirac equation

$$(\mathrm{i}\not{D} - m_\chi)\chi = 0 \tag{22}$$

holds.

Note that the effect of the existence of effective operators in the Lagrangian is not taken into account, which leads to additional terms suppressed by powers of the mediator scale Λ. The validity of these equations for our purposes is discussed in detail in 5.2.

4.3 Effective description below the electroweak scale

Below the electroweak scale, the Higgs field H acquires a vacuum expectation value v_{EW} by virtue of its potential $V(H)$, so that we need to perform a field redefinition to use standard perturbative methods. We set

$$H := \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v_{EW} + h + \mathrm{i}G^0) \end{pmatrix}, \tag{23}$$

where h is the Higgs boson and $G^{0,+}$ are Goldstone bosons.

However, the Higgs gauge interactions induce mass terms for the $SU(2)$ and $U(1)$ gauge bosons. In order to switch to the mass eigenbasis, we rotate by the Weinberg mixing angle ϑ_w. Additionally, we split up the fermion fields along the broken $SU(2)$ symmetry:

$$\begin{gathered} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} := \begin{pmatrix} c_w & -s_w \\ s_w & c_w \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}, \qquad \begin{pmatrix} W^+_\mu \\ W^-_\mu \end{pmatrix} := \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -\mathrm{i} \\ 1 & \mathrm{i} \end{pmatrix} \begin{pmatrix} W^1_\mu \\ W^2_\mu \end{pmatrix}, \\ \begin{pmatrix} u_L \\ d_L \end{pmatrix} := Q_L, \qquad \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} := L_L, \end{gathered} \tag{24}$$

where we have introduced the shorthands $s_w := \sin\vartheta_w$ and $c_w := \cos\vartheta_w$. Now, the gauge and Higgs boson masses are given by

$$m_{W^\pm} = g_2 \frac{v_{EW}}{2}, \qquad m_Z = \sqrt{g_1^2 + g_2^2}\frac{v_{EW}}{2}, \qquad m_h = v_{EW}\sqrt{\lambda/2}, \tag{25}$$

where we have used the Higgs quartic coupling, which is roughly of order one, to express the Higgs mass in terms of the electroweak scale. Considering this, we integrate out the heavy degrees of freedom $W^\pm, Z$ and H. For convenience, we also introduce the electric charge

$$e := \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} = g_2 s_w = g_1 c_w. \tag{26}$$

These redefinitions lead to the following covariant derivative:

$$D_\mu = \partial_\mu - \mathrm{i}g_3 T^a G^a_\mu - \mathrm{i}eQA_\mu - \frac{\mathrm{i}e}{s_w c_w}\left(\tau^3 - s_w^2 Q\right) Z_\mu - \frac{\mathrm{i}g_2}{\sqrt{2}}\left(\tau^+ W^+_\mu + \tau^- W^-_\mu\right), \tag{27}$$

where $\tau^\pm := \tau^1 \pm \mathrm{i}\tau^2$ and the Gell-Mann Nishijima relation $Q = \tau^3 + Y_\chi$ holds. For matching our operators onto the basis below the electroweak scale, where the rotated

$U(1)$ remains manifestly unbroken, it is useful to define a new photon field strength tensor by

$$F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{28}$$

This implies

$$\begin{aligned} W^3_{\mu\nu} &= s_w F_{\mu\nu} + c_w(\partial_\mu Z_\nu - \partial_\nu Z_\mu) + \mathrm{i} g_2 (W^-_\mu W^+_\nu - W^+_\mu W^-_\nu), \\ B_{\mu\nu} &= c_w F_{\mu\nu} - s_w(\partial_\mu Z_\nu - \partial_\nu Z_\mu), \end{aligned} \tag{29}$$

and the equation of motion

$$D^\mu F_{\mu\nu} = \partial^\mu F_{\mu\nu} = -e \sum_f Q_f \bar{f} \gamma_\nu f. \tag{30}$$

We also perform a chiral rotation of the χ field:

$$\chi \mapsto \exp\left(\frac{\mathrm{i}}{2}\gamma_5 \phi\right)\chi, \tag{31}$$

which will allow us to choose ϕ in a way to eliminate an imaginary mass term possibly induced by effective operators. This will be discussed in greater detail in chapter 6. Since the electrically charged components of the dark matter field decouple at the level of our discussion, we will use χ to refer to only the uncharged component of the dark matter field after breaking the electroweak symmetry.

4.3.1 Light dark matter

Assuming that the dark matter mass m_χ is small with respect to the electroweak scale, we can write the renormalizable part of its Lagrangian as

$$\mathcal{L}^{\mathrm{DM}} = \bar{\chi} \mathrm{i} \partial\!\!\!/ \chi, \tag{32}$$

with an additional customary factor 1/2 for the Majorana case. We find a full basis of operators for this theory in chapter 5 and discuss our notation of the effective Lagrangian before matching to the UV basis in chapter 6.

4.3.2 Electroweak-scale dark matter

For the case of a dark matter mass of roughly the electroweak scale, we employ the Heavy Dark Matter Effective Theory (HDMET, [27]), which is constructed analogously to Heavy Quark Effective Theory (see e.g. [28, 29]). The ansatz for this treatment is the fact that for direct detection experiments, the typical momentum exchanges are very small compared to the dark matter mass. Thus, we can introduce a four-velocity v^μ that we choose relativistically close to the dark matter velocity before or after the scattering, and write

$$p = m_\chi v + k \tag{33}$$

for the dark matter momentum p, where the *residual momentum* k is much smaller than m_χ. This will allow us to perform an expansion in the inverse dark matter mass $1/m_\chi$.

Using the projection operators

$$P_\pm = \frac{1 \pm \slashed{v}}{2}, \tag{34}$$

we split up the dark matter field χ into a *small-component field* χ_v and a *large-component field* X_v according to

$$\chi_v(x) = e^{\mathrm{i}m_\chi v\cdot x} P_+ \chi(x) \quad \text{and} \quad X_v(x) = e^{\mathrm{i}m_\chi v\cdot x} P_- \chi(x), \tag{35}$$

so that

$$\chi(x) = e^{-\mathrm{i}m_\chi v\cdot x} \left(\chi_v(x) + X_v(x)\right) \tag{36}$$

holds. Interpreting the relations $P_-\chi_v = P_+X_v = 0$ in the rest frame allows us to conclude that χ_v and X_v are associated with the particle and anti-particle modes of χ, respectively. This redefinition transforms the usual Dirac Lagrangian into

$$\mathcal{L}_v = \bar{\chi}_v \mathrm{i}v\cdot\partial\chi_v - \bar{X}_v(\mathrm{i}v\cdot\partial + 2m_\chi)X_v + \bar{\chi}_v \mathrm{i}\slashed{\partial}_\perp X_v + \bar{X}_v \mathrm{i}\slashed{\partial}_\perp \chi_v, \tag{37}$$

where we introduce the shorthand $X_\perp^\mu = X^\mu - v^\mu v\cdot X$ for any object X with a Lorentz index.

Since we need a momentum of order $\mathcal{O}(2m_\chi)$ to excite anti-particle modes, we integrate out X_v. Deriving and exploiting the coupled equations of motion from Eq. (37) allows us to write down an effective Lagrangian

$$\mathcal{L}_v^{\text{eff}} = \bar{\chi}_v \mathrm{i}v\cdot\partial\chi_v + \bar{\chi}_v \mathrm{i}\slashed{\partial}_\perp \frac{1}{2m_\chi + \mathrm{i}v\cdot\partial} \mathrm{i}\slashed{\partial}_\perp \chi_v. \tag{38}$$

Expanding in powers of $1/m_\chi$, we obtain our final expression for the Lagrangian

$$\mathcal{L}^{\text{HDMET}} = \bar{\chi}_v \mathrm{i}v\cdot\partial\chi_v + \frac{1}{2m_\chi}\bar{\chi}_v(\mathrm{i}\partial_\perp)^2\chi_v + \mathcal{O}(1/m_\chi^2), \tag{39}$$

where the equation of motion for χ_v is now given by

$$\mathrm{i}v\cdot\partial\chi_v = -\frac{1}{2m_\chi}(\mathrm{i}\partial_\perp)^2\chi_v + \mathcal{O}(1/m_\chi^2). \tag{40}$$

At tree-level and to first order in $1/m_\chi$, this procedure is equivalent to applying the following identity to the original Lagrangian:

$$\chi = e^{-\mathrm{i}m_\chi v\cdot x}\left(1 + \frac{\mathrm{i}\slashed{\partial}}{2m_\chi} + \mathcal{O}\left(\frac{1}{m_\chi^3}\right)\right)\chi_v. \tag{41}$$

The situation for the Majorana case is largely the same: The additional Majorana condition does not imply any relation between the remaining active degrees of freedom in HDMET, leading to the same Lagrangian as in the Dirac case. However, to obtain this canonically normalized Lagrangian, we have to insert an additional factor of $1/\sqrt{2}$ on the right hand side of the new fields definition in Eq. (35). This modifies the tree-level relation Eq. (41) by a factor $\sqrt{2}$. For more details on this, see Ref. [30].

Some useful identities for calculations in HDMET include

$$P_\pm\gamma^\mu P_\pm = \pm P_\pm v^\mu, \qquad P_\pm\gamma^\mu P_\mp = P_\pm\gamma_\perp^\mu, \tag{42}$$

$$P_\pm[\gamma^\mu,\gamma^\nu]P_\pm = P_\pm[\gamma_\perp^\mu,\gamma_\perp^\nu], \qquad P_\pm[\gamma^\mu,\gamma^\nu]P_\mp = 4P_\pm v^{[\mu}\gamma_\perp^{\nu]}. \tag{43}$$

We discuss the effective Lagrangian for this theory before performing the matching to the full UV basis in chapter 6.

4.4 Effective description at low energies

While we have yet to extend the treatment of [2] for our higher-dimensional operators, this section sketches the remaining steps to connect the UV Wilson coefficients to the differential DM-nucleus interaction cross section as the primary observable in direct detection experiments.

Chiral Perturbation Theory

The starting point of chiral perturbation theory is to inspect the symmetries of the QCD Lagrangian just above the non-perturbative regime to constrain our effective description at lower energies. Since only three quarks remain as active degrees of freedom, we find that, disregarding the small quark masses and effective operators with (pseudo-)scalar and tensor quark currents, it is invariant under global chiral rotations in flavour space:

$$U_L(3) \times U_R(3) = SU_L(3) \times SU_R(3) \times U_V(1) \times U_A(1). \tag{44}$$

It turns out that non-perturbative quantum effects induce a non-zero quark condensate, spontaneously breaking the approximate $SU_L(3) \times SU_R(3)$ down to $SU_V(3)$, the subgroup where left- and right-handed quarks transform under the same rotation. The eight pseudoscalar mesons can be seen as the pseudo-Nambu Goldstone bosons resulting from breaking the approximate $(SU_L(3) \times SU_R(3))/SU_V(3) \cong SU(3)$.

The quark masses and effective operators, of course, explicitly break all these symmetries. To systematically include them in our effective theory, we perform a *spurion analysis*. As far as QCD is concerned, dark matter currents can be treated as external fields, so that we rewrite the QCD part of the Lagrangian in terms of spurions $s_g, \theta, v_\mu, a_\mu, s, p$ and $t_{\mu\nu}$:

$$\begin{aligned}\mathcal{L}^{\mathrm{QCD}} = \mathcal{L}_0^{\mathrm{QCD}} &+ s_G(x) G^a_{\mu\nu} G_a^{\mu\nu} + \theta(x) G^a_{\mu\nu} \tilde{G}_a^{\mu\nu} \\ &+ \bar{q}(x) \gamma^\mu \left(v_\mu(x) + \gamma_5 a_\mu(x)\right) q(x) \\ &- \bar{q}(x) \left(s(x) - \mathrm{i}\gamma_5 p(x)\right) q(x) \\ &+ \bar{q}(x) \sigma^{\mu\nu} t_{\mu\nu}(x) q(x),\end{aligned} \tag{45}$$

where $\mathcal{L}_0^{\mathrm{QCD}}$ contains the kinetic terms of quarks and their QED interactions. We need to add the tensor current with respect to [2], since our operators $\mathcal{Q}_{9,q}^{(2F)}$ and $\mathcal{Q}_{10,q}^{(2F)}$ now contribute to it.

Spurion analysis is based on the principle that the form chiral symmetry breaking takes should be barely dependent of the details of the physics encoded in the spurions, since one can expand in spurion insertions if they only weakly break the symmetry. Therefore, we can introduce an artificial local $SU_L(3) \times SU_R(3)$ symmetry acting as

$$q(x) \mapsto V_R(x)\, P_R\, q(x) + V_L(x)\, P_L\, q(x) \tag{46}$$

on the quarks and freely choose the transformation properties of the spurions in a way to leave the Lagrangian invariant:

$$s + \mathrm{i}p \mapsto V_R (s + \mathrm{i}p) V_L^\dagger \tag{47}$$

$$
\begin{aligned}
s_G &\mapsto s_G && (48)\\
v_\mu + a_\mu &\mapsto V_R(v_\mu + a_\mu)V_R^\dagger + \mathrm{i}V_R\partial_\mu V_R^\dagger && (49)\\
v_\mu - a_\mu &\mapsto V_L(v_\mu - a_\mu)V_L^\dagger + \mathrm{i}V_L\partial_\mu V_L^\dagger && (50)\\
t_{\mu\nu} &\mapsto \frac{1}{2}V_R\left(t_{\mu\nu} + \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}t^{\alpha\beta}\right)V_L^\dagger + \frac{1}{2}V_L\left(t_{\mu\nu} - \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}t^{\alpha\beta}\right)V_R^\dagger && (51)
\end{aligned}
$$

Now, we can go on to construct our effective theory, building a Lagrangian respecting our artificial symmetry and using the spurions, light pseudoscalars and baryons as degrees of freedom. Due to their high mass, the baryons get treated as heavy analogously to Heavy Quark Effective Theory or HDMET. As a counting scheme, the usual expansion in momenta and masses is employed.

The Wilson coefficients in this context are often called *low-energy constants* or *Gasser-Leutwyler coefficients* after the original authors who derived the ChPT Lagrangian [31]. For numerical calculations, additional input is needed to fix some of these coefficients in the form of non-perturbative observables such as the quark condensate and pion decay constant. These can be obtained either from experiments or lattice computations.

Chiral counting in ChEFT

The description of forces at a nuclear scale that is outlined above breaks down for nuclei due to the appearance of reducible infrared-divergent diagrams, i.e. those with only nucleons in intermediate states. Weinberg solved this problem: One can recover N-nucleon amplitudes by solving the Lippmann-Schwinger equation with an effective potential constructed from irreducible diagrams, effectively resumming the reducible diagrams [32, 33].

For a more thorough discussion, see [2] or [34].

Effective theory of nuclear response

In a last step, the theory of nuclear response in direct detection searches as developed in [35] is used to derive the DM-nucleus cross section. To this end, we need to match the description of the last section onto the Wilson coefficients of the non-relativistic operators describing the scattering of dark matter off protons and neutrons.

For a nucleus of mass m_A and spin J_A, this yields

$$
\frac{\mathrm{d}\sigma}{\mathrm{d}E_R} = \frac{m_A}{4\pi|v_\chi|^2}\,\frac{1}{2J_A+1}\,\sum_{\text{spins}}|\mathcal{M}|^2_{\text{NR}}, \tag{52}
$$

where the squared non-relativistic matrix element can be found as an explicit expression of Wilson coefficients weighing nuclear response functions.

5 Construction of the operator basis

As an extension of the dimension five and six basis for effective operators with fermionic dark matter fields constructed in the upcoming paper by Bishara, Brod, Grinstein and Zupan [1], we have constructed an analogous basis at dimension seven. We automated this task through custom C++ code generating all expressions invariant under standard model symmetries to consistently exploit a range of linear relations between operators. Our program replicates the results of the lower-dimensional operator bases in [1], and was compared with the numbers of independent operators calculated using Hilbert series methods with code supplied with [36].

As stated previously, we limit our analysis to operators containing an even number of dark matter fields. The rationale behind this is not to introduce interactions that would make our dark matter unstable. From the perspective of a UV completion, this could typically be implemented using a $U(1)$ or $\mathcal{Z}_2$ symmetry. We only find independent operators with exactly two χ fields at dimension seven.

We give only operators without charge-conjugated dark matter fields. In the Majorana case, no additional operators exist, while in the Dirac case we obtain a copy of our list with $\bar{\chi}\chi \to \bar{\chi}^c\chi$, since $Y_\chi = 0$. If we were to relax this assumption, which of course is only possible in the Dirac case, the list with $\bar{\chi}\chi$ currents remains the same, while the operators with charge-conjugated dark matter fields now heavily on the chosen value of Y_χ. For $Y_\chi = \pm 1/2$ and ± 1, we would induce additional operators starting at dimensions five and six, respectively. Any other assignment of Y_χ would either prevent an electrically neutral component of χ or not generate any new operators up to dimension seven.

Our algorithm is independent of the chosen $SU(2)$-representation of χ by treating it as different from all SM representations. Thereby, we construct a superset of bases for all possible representations. Additional reduction identities that are relevant for specific representations of $SU(2)$ and for Majorana particles are discussed in Section 5.2.

The reason why no new operators with an even number of χ fields can come up in specific representations is the following: The only new covariant symbol available for contraction is the Levi-Civita symbol for that representation. For the fundamental and adjoint representation, these are already considered in our operator list, and for any higher-dimensional representation, we cannot form a singlet using only this symbol (in an odd-dimensional representation, as argued in Section 4.1) and the even number of indices from the dark matter fields. Operators where the $SU(2)$ indices of χ are contracted with other fields can always be rearranged to operators in our list using Fierz identities.

In this chapter, we will first outline our algorithm and then go on to discuss all relations that were exploited to remove linearly dependent operators and the consistency checks we performed. Finally, we close by discussing the resulting basis, both above and below the electroweak scale.

5.1 Steps of the algorithm

The following steps are executed for the construction of an operator basis:

1. Construction of all possible hypercharge-conserving operator classes at the desired mass dimension

2. Generation of all possible primary index contractions for every operator class

3. Addition of Levi-Civita symbols

4. Contraction of all remaining indices

5. Finding hermitian combinations and removal of superfluous operators

For our purposes, an *operator class* is specified by the combination of field operators and covariant derivatives that constitute the operator. For example, one such class would be $D^2[\bar{\chi}]\chi A$. The list of allowed operator classes is easily constructed by exhausting all combinations of fields and derivatives up to a given mass dimension.

In the second step, we regard the operator classes as possible terms that can appear in operators, whose indices have not yet been contracted. By *primary indices* we denote pairs of indices for which two things hold: Firstly, we can specify a complete basis of all covariant matrices between them, possibly with additional attached indices. Secondly, these indices cannot be introduced when contracting other primary indices (thereby guaranteeing this step of the computation to terminate). The typical example is the covariant basis of matrices between Dirac-spinors, but in our case, all indices except those of the Lorentz group and adjoint representations are primary, since these two kinds of indices appear in matrices that contract other indices like γ^μ and τ^a. For each class, we generate all possible contractions of primary indices using their complete bases.

For each candidate operator constructed up to now, we add more candidates with any combination of Levi-Civita symbols for Lorentz and adjoint $SU(2)$ indices in the third step. We do not consider $SU(3)$ or fundamental $SU(2)$ symbols, since the former cannot appear with the maximum of two colored particles that come up in our operator class list, and the latter is already included in the primary contractions. Furthermore, we do not add more than one for each type of index, since (for the real representations considered here) a product of two epsilon tensors can be reexpressed with Kronecker deltas by utilising their representation as determinants, so that these operators are not independent from those already constructed.

In step 4, we construct all possible contractions of the remaining open indices, with the exception of indices not associated with an unbroken symmetry. For our situation, this is only the case for standard model generation indices. We discard all operators whose indices cannot be fully contracted.

Finally, we find hermitian combinations of the complete set of terms we have generated thus far and reduce linearly dependent operators, as discussed in the following section.

5.2 Reduction identities

After writing down all possible invariant operators, the list needs to be reduced to a linearly independent set. The linear relations fall into three categories: Equations of motion, relations only valid in four spacetime dimensions (usually referred to as 'evanescent' in the literature) as well as relations valid in all spacetime dimensions. All of these can be consistently used in dimensional regularisation to reduce the operator basis before calculating the renormalization group running, as H. Simma discusses in [37].

To keep track of all generated relations, we add them as rows to a matrix M, so that, with a vector $\vec{Q}$ of operators, $M\vec{Q} = 0$ holds. To find an independent set of operators, we now put M into reduced row echelon form. Going through the matrix row-by-row, we can always disregard the first operator with a non-zero coefficient. This holds, because if it is the only coefficient in that row with a non-zero coefficient, it must vanish, but if there are other operators in that row, we can reexpress it with those that come after it. In order to preferentially keep operators that can be expressed in a simple way, we order the operators and thereby the columns of the matrix before the reduction, giving preference to self-hermitian terms, with the fewest number of Levi-Civitas, simplest Dirac matrices and rightmost covariant derivatives, in this order. This procedure was already used by Gripaios and Sutherland in [38].

The rest of this section will go through all identities exploited by our program.

Permutations of identical fields and index symmetries To remove all possible duplicate operators, we explicitly check the disparity of operators under simultaneous permutations of (anti-)symmetric indices and index sets of identical fields, keeping track of fermionic signs. This also allows us to identify operators that vanish already per symmetries.

Chirality of standard model fermions Since we work in four-component notation, we need to eliminate all operators that vanish via the chirality of the respective fields and simplify expressions containing γ_5 that arise in Fierz and EOM relations. The identity

$$\sigma^{\mu\nu}\mathrm{i}\gamma_5 = -\frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta} \tag{53}$$

is especially useful in this regard.

De-facto symmetry of covariant derivatives Pairs of covariant derivatives acting on the same field can be decomposed into parts that are symmetric and antisymmetric. Since the antisymmetric part is given by the commutator $[D_\mu, D_\nu]$, which is in turn given by the *Ricci identity* as a sum of the gauge field strength tensors and the appropriate generators, it is already covered by operators with a lower count of covariant derivatives, which is guaranteed to be in a distinct class. Therefore, our program treats covariant derivatives as if they were symmetric in their indices.

Integration by parts Since total derivatives have no physical effects, expanding them using the Leibniz rule yields additional relations between our operators. We keep our list manifestly covariant by using covariant derivatives, which are of course constructed to also satisfy the Leibniz rule. Algorithmically, we implement this by constructing all operators with one open Lorentz index at mass dimension six and subsequently contracting the open index with the total derivative of this operator.

Fierz identities Fierz identities are a general consequence of completeness relations in a given vector space and can be used to rearrange the fields in pairs of bilinears; for a general discussion and derivation see e.g. [39]. The general form of the well-known Fierz identities allows us to disregard primary contractions that are not in an (arbitrarily chosen) standard order, since any operator obtained by reordering the pattern of contraction will be a linear combination of the previous operators. This argument also applies to more than two bilinears, since a general permutation of fields can be expressed as a number of transpositions, for which the usual formula holds.

While the Fierz relations can be used to move all bilinears into a standard ordering, there are two cases where they imply additional relations: Firstly, if two fields in the bilinears are identical, using them to swap these fields directly relates terms that are already in standard order. Secondly, if an expression in non-standard order vanishes, e.g. by chirality, EOM's or IBP identities, then the equivalent expression in standard order must vanish too, possibly yielding a new, independent relation.

The classical example of Fierz relations considers the standard covariant basis of Dirac bilinears,

$$\{\Gamma^A\} = \{1, \gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, \sigma^{\mu\nu}\}, \tag{54}$$

where $\nu < \mu = 0...3$, and its dual basis constructed with respect to the Frobenius scalar product, so that

$$\mathrm{Tr}(\Gamma_A \Gamma^B) = 4\delta_A^B. \tag{55}$$

It posits that

$$\Gamma^A_{ij}\Gamma^B_{kl} = \frac{1}{16}\,\mathrm{Tr}(\Gamma^A\Gamma_C\Gamma^B\Gamma_D)\;\Gamma^D_{il}\Gamma^C_{kj}. \tag{56}$$

We generated all explicit Fierz identities from this formula using FORM, since usually only those that relate Lorentz singlets are given in the literature. We then implemented C++ code to treat only those cases that could be used to reduce our list of operators, but it turned out that all additional relations generated this way were already implied by Eq. (53), which can be used to move $\mathrm{i}\gamma_5$ into the SM current and subsequently exploit the fields chirality.

For the fundamental representation of $SU(2)$, the following explicit Fierz identities were used:

$$
\begin{aligned}
(1)_{ij}(1)_{kl} &= 2(\tau^a)_{il}(\tau^a)_{kj} + \frac{1}{2}(1)_{il}(1)_{kj} \\
(\tau^a)_{ij}(1)_{kl} &= \mathrm{i}\varepsilon^{abc}(\tau^c)_{il}(\tau^b)_{kj} + \frac{1}{2}(\tau^a)_{il}(1)_{kj} + \frac{1}{2}(1)_{il}(\tau^a)_{kj} \\
(\tau^a)_{ij}(\tau^b)_{kl} &= \delta^{ab}\left(\frac{1}{4}(1)_{il}(1)_{kj} - (\tau^c)_{il}(\tau^c)_{kj}\right) + \frac{1}{2}\left((\tau^a)_{il}(\tau^b)_{kj} + (\tau^b)_{il}(\tau^a)_{kj}\right) \\
&\quad + \frac{\mathrm{i}}{4}\varepsilon^{abc}\left((\tau^c)_{il}(1)_{kj} - (1)_{il}(\tau^c)_{kj}\right),
\end{aligned}
\tag{57}
$$

where the latter two equations can easily be derived from the well-known first relation by using

$$
\tau^a\tau^b = \frac{\mathrm{i}}{2}\varepsilon^{abc}\tau^c + \frac{1}{4}\delta^{ab}. \tag{58}
$$

If we assume the general case that χ is in a different representation of $SU(2)$ than all SM fields and since, judging from the possible operator classes enumerated in Sec. 5.4 and 5.5, there cannot be more than one SU(3) bilinear, these are all the Fierz relations we can make use of.

Products of Levi-Civita symbols As previously stated, we already exploited the fact that we can reduce products of Levi-Civita symbols to sums of Kronecker deltas by not considering such operators in the first place, our treatment of Fierz relations and EOMs reintroduce such products. For Lorentz indices, for example, we then use the general formula

$$
\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{\lambda\rho\sigma\tau} = -\begin{vmatrix}
\delta^\lambda_\mu & \delta^\lambda_\nu & \delta^\lambda_\alpha & \delta^\lambda_\beta \\
\delta^\rho_\mu & \delta^\rho_\nu & \delta^\rho_\alpha & \delta^\rho_\beta \\
\delta^\sigma_\mu & \delta^\sigma_\nu & \delta^\sigma_\alpha & \delta^\sigma_\beta \\
\delta^\tau_\mu & \delta^\tau_\nu & \delta^\tau_\alpha & \delta^\tau_\beta
\end{vmatrix} \tag{59}
$$

to express these relations through the operators on our list, where the vertical bars denote the usual matrix determinant. This also implies the well-known formulae

$$
\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{\lambda\rho\alpha\beta} = -4\delta^\lambda_{[\mu}\delta^\rho_{\nu]}, \quad \varepsilon_{\mu\nu\alpha\beta}\varepsilon^{\lambda\nu\alpha\beta} = -6\delta^\lambda_\mu, \quad \varepsilon_{\mu\nu\alpha\beta}\varepsilon^{\mu\nu\alpha\beta} = -24, \tag{60}
$$

where square brackets denote antisymmetrization.

Schouten identities Another useful identity to find relations between operators containing Levi-Civita symbols are *Schouten identities*. These follow from the fact that there is no totally antisymmetric tensor with more indices than the vector space dimension other than the zero tensor. E.g., antisymmetrizing an object with more than four Lorentz indices yields zero. In the literature, this is usually expressed by noting that for any 4-vector V_μ,

$$
V_\mu\varepsilon_{\alpha\beta\gamma\delta} - V_\alpha\varepsilon_{\mu\beta\gamma\delta} - V_\beta\varepsilon_{\alpha\mu\gamma\delta} - V_\gamma\varepsilon_{\alpha\beta\mu\delta} - V_\delta\varepsilon_{\alpha\beta\gamma\mu} = 0 \tag{61}
$$

holds.

These identities were applied semi-manually to eliminate six operators in the Gauge-Gauge operator class. For example, antisymmetrizing the indices of the Levi-Civita symbol together with μ in

$$(\bar{\chi}\sigma_{\mu\nu}\chi)B^{\mu}{}_{\sigma}B_{\alpha\beta}\varepsilon^{\nu\sigma\alpha\beta}. \tag{62}$$

and dropping terms that vanish by symmetry implies that the whole operator vanishes. Moreover, Schouten identities can also yield relations like

$$\bar{\chi}\sigma_{\mu\nu}\tau^a\chi\, W^b_{\sigma}{}^{\nu}W^c_{\alpha\beta}\,\varepsilon^{abc}\varepsilon^{\mu\sigma\alpha\beta} = \bar{\chi}\sigma_{\mu\nu}\tau^a\chi\, W^b_{\sigma}{}^{\alpha}W^c_{\alpha\beta}\,\varepsilon^{abc}\varepsilon^{\mu\sigma\nu\beta}, \tag{63}$$

which follows from antisymmetrizing the Levi-Civita symbol with ν on the left-hand side. Since the naïve algorithm antisymmetrizing all sufficiently large sets of indices is very computationally demanding, index sets for a small number of operators were specified by hand.

Bianchi identity The Jacobi identity of the Lie-bracket together with the Ricci identity implies the *Bianchi identity*, which for any gauge field strength tensor $G^a_{\mu\nu}$ is given by

$$D_\mu G^a_{\nu\rho} + D_\nu G^a_{\rho\mu} + D_\rho G^a_{\mu\nu} = 0. \tag{64}$$

In particular, this implies that the covariant derivative of the dual tensor vanishes:

$$D^\mu \tilde{G}^a_{\mu\nu} = 0, \quad \text{where} \quad \tilde{G}^a_{\mu\nu} := \frac{1}{2}\varepsilon_{\mu\nu\rho\eta}G^{\rho\eta}_a. \tag{65}$$

We implement this together with our treatment of equations of motion for gauge tensors due to their similar form.

Additional reduction for special representations of $SU(2)$ While this operator basis is valid for any irreducible representation of $SU(2)$ that the dark matter field χ assumes, additional relations can make some operators superfluous in specific representations.

For the trivial representation of $SU(2)$, it suffices to drop all operators with a $SU(2)$ generator in the dark matter current. For the fundamental representation, we find no additional relations that lead to redundant operators. For the adjoint representation, the generator is given by the structure constant, which is just the Levi-Civita symbol. Therefore, operators with both a generator and an additional Levi-Civita symbol can be simplified as discussed above.

Additional reduction for the Majorana case For every dark matter Dirac bilinear, we can use the Majorana condition to replace χ with its charge conjugate. Simplifying the resulting operator yields the negative of the original operator for the Dirac matrices γ^μ and $\sigma^{\mu\nu}$ in the bilinear, so that such operators must vanish. In other words, every dark matter bilinear that by itself is odd under charge symmetry can be dropped from the list in the Majorana case, eliminating DM vector and tensor currents.

Note that an antisymmetric covariant derivative would affect this argument if it acts within the bilinear. However, we chose our basis in such a way that all derivatives act within the standard model part of each operator.

For the Majorana case, we implicitly include a customary factor 1/2 for every dark matter bilinear in the basis, in order to simplify a simultaneous treatment of both types of fermions.

5.2.1 Equations of motion

For scalar fields and field strength tensors, the reduction with equations of motion is fairly straightforward. For Dirac fields, however, the Clifford algebra complicates matters. To deal with this, we first construct a new set of only EOM-vanishing operators by finding all operators of the appropriate mass dimension with any of the following matrix expressions in Dirac field bilinears:

$$\{\gamma^\mu, \gamma^\mu\gamma_5, \gamma^\mu\gamma^\nu, \gamma^\mu\gamma^\nu\gamma_5, \gamma^\mu\gamma^\nu\gamma^\rho, \gamma^\mu\gamma^\nu\gamma^\rho\gamma_5\}, \tag{66}$$

where either the rightmost or leftmost Lorentz index is contracted with a covariant derivative acting on the right or left Dirac field, respectively. The remaining open indices as well as the remaining fields are contracted with the same algorithm used for the main operator list to get all covariant expressions containing this EOM-vanishing bilinear. We then proceed to translate each of these into relations between our full list by exploiting the identities

$$\gamma_\mu\gamma_\nu = \eta_{\mu\nu} - \mathrm{i}\sigma_{\mu\nu}, \tag{67}$$
$$\gamma_\mu\gamma_\nu\gamma_5 = \eta_{\mu\nu}\gamma_5 - \mathrm{i}\sigma_{\mu\nu}\gamma_5, \tag{68}$$
$$\gamma_\mu\gamma_\nu\gamma_\rho = \eta_{\mu\nu}\gamma_\rho + \eta_{\nu\rho}\gamma_\mu - \eta_{\mu\rho}\gamma_\nu - \mathrm{i}\varepsilon_{\sigma\mu\nu\rho}\gamma^\sigma\gamma_5, \tag{69}$$
$$\gamma_\mu\gamma_\nu\gamma_\rho\gamma_5 = \eta_{\mu\nu}\gamma_\rho\gamma_5 + \eta_{\nu\rho}\gamma_\mu\gamma_5 - \eta_{\mu\rho}\gamma_\nu\gamma_5 - \mathrm{i}\varepsilon_{\sigma\mu\nu\rho}\gamma^\sigma, \tag{70}$$

where the chirality of standard model fields is used to absorb γ_5, if possible. If there already exists a Lorentz Levi-Civita symbol in the remainder of the operator, their product gets simplifies as described above.

We argue that this procedure (only together with the previously discussed treatment of partial integration and index symmetries, of course) captures all relations implied by equations of motion. To see this, consider a completely general EOM-vanishing operator

$$\bar{X}M\gamma^\mu D_\mu X..., \tag{71}$$

where X is any Dirac field, M is any sum of products of Dirac matrices, possibly contracted with other expressions in the remainder of the operator, which is denoted by dots. The case where the covariant derivative acts on the left or additional derivatives appear can be treated analogously in the following argument. This operator, when expressed in our basis, does not induce any relations not covered by the above procedure, since Eq. (69) and (70) can be repeatedly applied within each term of M to reduce it to a sum of terms with a maximum of two gamma matrices (and possibly one γ_5, which can always be anticommuted to the rightmost position and eliminated if an even number of them exists), so that we have expressed this operator as a linear combination of the vanishing operators already constructed by our program. The relation it implies, therefore, is also linearly dependent.

5.2.2 Construction of hermitian combinations

Since the discussed relations contain complex coefficients and mostly relate single terms, we find it most convenient to implement a complex matrix representing these relations before finding hermitian combinations. In step 5, however, we convert this to a real matrix by first doubling our number of terms, multiplying half of them with i, and then appropriately building purely real relations from each original relation as well as its complex conjugate. We then find pairs of expressions related by hermitian conjugation (keeping track of permutation and fermionic signs) and construct an invertible matrix H, which relates a vector $\vec{Q}$ of the original operators to a new vector $\vec{Q}' = H\vec{Q}$, consisting of sums and differences of terms connected by conjugation and keeping terms that are already (anti-)hermitian intact. The new matrix that we need to put into row echelon form, as discussed in the introduction to this section, is then given by $M' = MH^{-1}$. After removing redundant operators we also drop all antihermitian combinations as unphysical.

5.3 Consistency checks

Optimally, we would have liked to reproduce the well-investigated standard model operators basis at dimension six, first discussed by Buchmüller and Wyler in [40] and revisited by Grzadkowski et al. in [41]. However, since we have mostly exploited Fierz relations either by hand or in a way that is restricted to our basis, we deemed this impracticable. But, as stated previously, we are able to reproduce the basis for our effective dark matter setup at dimensions five and six without any manual intervention.

Additionally, we compared our basis with operator counts for a given field content that were derived using the conformal *Hilbert series method* [36, 42, 43]. This recent approach allows for a systematic group-theoretic treatment of equations of motion and integration by parts identities. It calculates an object called the Hilbert series, which is given by

$$H(\mathcal{D}, \phi_1, \ldots, \phi_N) = \sum_{k, r_1, \ldots, r_N} c_{k, r_1, \ldots, r_N}\ \phi_1^{r_1} \ldots \phi_N^{r_N}\ \mathcal{D}^k, \tag{72}$$

where ϕ_i and $\mathcal{D}$ are complex numbers that stand in for the fields of the theory and the covariant derivative. The coefficients $c_{k, r_1, \ldots, r_N}$ are the sought-after operator counts.

Michele Tammaro of the University of Cincinnati adapted the Mathematica code supplied with [36] to our scenario, which we found to be entirely consistent with our list.

5.4 The full UV basis

At dimension seven, we find the following types of operator classes that conserve hypercharge:

- $Q^{(GG)} \propto \bar{\chi}\chi\ G\ G$ - Gauge-Gauge
- $Q^{(GH)} \propto \bar{\chi}\chi\ G\ H^\dagger H$ - Gauge-Higgs
- $Q^{(Y)} \propto \bar{\chi}\chi\ \bar{Q}UH$ - Yukawa-like

- $Q^{(4H)} \propto \bar{\chi}\chi\ H^\dagger H\ H^\dagger H$ - Four-Higgs
- $Q^{(d2F)} \propto D\ \bar{\chi}\chi\ \bar{\Psi}\Psi$ - Four-Fermion
- $Q^{(2d2H)} \propto DD\ \bar{\chi}\chi\ H^\dagger H$ - Two-Higgs
- $Q^{(2dG)} \propto DD\ \bar{\chi}\chi\ G$ - Dipole-like (all vanish)
- $Q^{(4d)} \propto DDDD\ \bar{\chi}\chi$ - Four-Derivative (all vanish)

The number of independent, hermitian operators (with real-valued coefficients) for every special case are listed in Table II, where we used the additional reductions described in Section 5.2.

We now list all operators for every given class, using the shorthand $\overleftrightarrow{D}_\mu := \overleftarrow{D}_\mu - D_\mu$, and the convention that derivatives act until the end of a bracket or on the closest bracket. As a reminder, we define $\tilde{G}^a_{\mu\nu} := \frac{1}{2}\varepsilon_{\mu\nu\rho\eta}G^{\rho\eta}_a$ for field strength tensors, and for the Majorana case add a factor 1/2 for every dark matter bilinear.

Gauge-Gauge operators

$$Q^{(GG)}_1 = (\bar{\chi}\chi)B_{\mu\nu}B_{\mu\nu} \qquad Q^{(GG)}_2 = (\bar{\chi}\mathrm{i}\gamma_5\chi)B_{\mu\nu}B_{\mu\nu} \tag{73}$$
$$Q^{(GG)}_3 = (\bar{\chi}\chi)B_{\mu\nu}\tilde{B}_{\mu\nu} \qquad Q^{(GG)}_4 = (\bar{\chi}\mathrm{i}\gamma_5\chi)B_{\mu\nu}\tilde{B}_{\mu\nu} \tag{74}$$
$$Q^{(GG)}_5 = (\bar{\chi}\tau^a\chi)W^a_{\mu\nu}B_{\mu\nu} \qquad Q^{(GG)}_6 = (\bar{\chi}\mathrm{i}\gamma_5\tau^a\chi)W^a_{\mu\nu}B_{\mu\nu} \tag{75}$$
$$Q^{(GG)}_7 = (\bar{\chi}\tau^a\chi)W^a_{\mu\nu}\tilde{B}_{\mu\nu} \qquad Q^{(GG)}_8 = (\bar{\chi}\mathrm{i}\gamma_5\tau^a\chi)W^a_{\mu\nu}\tilde{B}_{\mu\nu} \tag{76}$$
$$Q^{(GG)}_9 = (\bar{\chi}\sigma_{\mu\nu}\tau^a\chi)W^a_{\mu\sigma}B_{\nu\sigma} \qquad Q^{(GG)}_{10} = (\bar{\chi}\sigma_{\mu\nu}\mathrm{i}\gamma_5\tau^a\chi)W^a_{\mu\sigma}B_{\nu\sigma} \tag{77}$$
$$Q^{(GG)}_{11} = (\bar{\chi}\chi)G^a_{\mu\nu}G^a_{\mu\nu} \qquad Q^{(GG)}_{12} = (\bar{\chi}\mathrm{i}\gamma_5\chi)G^a_{\mu\nu}G^a_{\mu\nu} \tag{78}$$
$$Q^{(GG)}_{13} = (\bar{\chi}\chi)G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} \qquad Q^{(GG)}_{14} = (\bar{\chi}\mathrm{i}\gamma_5\chi)G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} \tag{79}$$
$$Q^{(GG)}_{15} = (\bar{\chi}\chi)W^a_{\mu\nu}W^a_{\mu\nu} \qquad Q^{(GG)}_{16} = (\bar{\chi}\mathrm{i}\gamma_5\chi)W^a_{\mu\nu}W^a_{\mu\nu} \tag{80}$$
$$Q^{(GG)}_{17} = (\bar{\chi}\chi)W^a_{\mu\nu}\tilde{W}^a_{\mu\nu} \qquad Q^{(GG)}_{18} = (\bar{\chi}\mathrm{i}\gamma_5\chi)W^a_{\mu\nu}\tilde{W}^a_{\mu\nu} \tag{81}$$
$$Q^{(GG)}_{19} = (\bar{\chi}\sigma_{\mu\nu}\tau^a\chi)W^b_{\mu\sigma}W^c_{\nu\sigma}\epsilon^{abc} \qquad Q^{(GG)}_{20} = (\bar{\chi}\sigma_{\mu\nu}\mathrm{i}\gamma_5\tau^a\chi)W^b_{\mu\sigma}W^c_{\nu\sigma}\epsilon^{abc} \tag{82}$$

	Dirac DM			Majorana DM		
	Trivial	Fun.	Adj.	Trivial	Fun.	Adj.
CP-even	25	46	44	15	24	24
Total	50	92	88	30	48	48

TABLE II. Number of independent operators for every special case discussed in Section 5.2. Note that the basis for the fundamental representation is a superset for general representations and that additionally allowing operators with charge-conjugated dark matter doubles the operator count in the Dirac case.

Gauge-Higgs operators

$$Q_1^{(GH)} = (\bar{\chi}\sigma^{\mu\nu}\chi)B_{\mu\nu}\ H^\dagger H \qquad Q_2^{(GH)} = (\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\chi)B_{\mu\nu}\ H^\dagger H \tag{83}$$
$$Q_3^{(GH)} = (\bar{\chi}\sigma^{\mu\nu}\tau^a\chi)B_{\mu\nu}\ H^\dagger\tau^a H \qquad Q_4^{(GH)} = (\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\tau^a\chi)B_{\mu\nu}\ H^\dagger\tau^a H \tag{84}$$
$$Q_5^{(GH)} = (\bar{\chi}\sigma^{\mu\nu}\chi)W^a_{\mu\nu}\ H^\dagger\tau^a H \qquad Q_6^{(GH)} = (\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\chi)W^a_{\mu\nu}\ H^\dagger\tau^a H \tag{85}$$
$$Q_7^{(GH)} = (\bar{\chi}\sigma^{\mu\nu}\tau^a\chi)W^a_{\mu\nu}\ H^\dagger H \qquad Q_8^{(GH)} = (\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\tau^a\chi)W^a_{\mu\nu}\ H^\dagger H \tag{86}$$
$$Q_9^{(GH)} = (\bar{\chi}\sigma^{\mu\nu}\tau^a\chi)A^b_{\mu\nu}\ H^\dagger\tau^c H\epsilon^{abc} \qquad Q_{10}^{(GH)} = (\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\tau^a\chi)A^b_{\mu\nu}\ H^\dagger\tau^c H\epsilon^{abc} \tag{87}$$

Yukawa-like operators

$$Q_1^{(Y)} = (\bar{\chi}\chi)(\bar{L}EH + \text{h.c.}) \qquad Q_2^{(Y)} = (\bar{\chi}\mathrm{i}\gamma_5\chi)(\bar{L}EH + \text{h.c.}) \tag{88}$$
$$Q_3^{(Y)} = (\bar{\chi}\chi)(\mathrm{i}\bar{L}EH + \text{h.c.}) \qquad Q_4^{(Y)} = (\bar{\chi}\mathrm{i}\gamma_5\chi)(\mathrm{i}\bar{L}EH + \text{h.c.}) \tag{89}$$
$$Q_5^{(Y)} = (\bar{\chi}\tau^a\chi)(\bar{L}E\tau^a H + \text{h.c.}) \qquad Q_6^{(Y)} = (\bar{\chi}\mathrm{i}\gamma_5\tau^a\chi)(\bar{L}E\tau^a H + \text{h.c.}) \tag{90}$$
$$Q_7^{(Y)} = (\bar{\chi}\tau^a\chi)(\mathrm{i}\bar{L}E\tau^a H + \text{h.c.}) \qquad Q_8^{(Y)} = (\bar{\chi}\mathrm{i}\gamma_5\tau^a\chi)(\mathrm{i}\bar{L}E\tau^a H + \text{h.c.}) \tag{91}$$
$$Q_9^{(Y)} = (\bar{\chi}\sigma_{\mu\nu}\chi)(\bar{L}\sigma^{\mu\nu}EH + \text{h.c.}) \qquad Q_{10}^{(Y)} = (\bar{\chi}\sigma_{\mu\nu}\chi)(\mathrm{i}\bar{L}\sigma^{\mu\nu}EH + \text{h.c.}) \tag{92}$$
$$Q_{11}^{(Y)} = (\bar{\chi}\sigma_{\mu\nu}\tau^a\chi)(\bar{L}\sigma^{\mu\nu}E\tau^a H + \text{h.c.}) \qquad Q_{12}^{(Y)} = (\bar{\chi}\sigma_{\mu\nu}\tau^a\chi)(\mathrm{i}\bar{L}\sigma^{\mu\nu}E\tau^a H + \text{h.c.}) \tag{93}$$
$$Q_{13}^{(Y)} = (\bar{\chi}\chi)(\bar{Q}DH + \text{h.c.}) \qquad Q_{14}^{(Y)} = (\bar{\chi}\mathrm{i}\gamma_5\chi)(\bar{Q}DH + \text{h.c.}) \tag{94}$$
$$Q_{15}^{(Y)} = (\bar{\chi}\chi)(\mathrm{i}\bar{Q}DH + \text{h.c.}) \qquad Q_{16}^{(Y)} = (\bar{\chi}\mathrm{i}\gamma_5\chi)(\mathrm{i}\bar{Q}DH + \text{h.c.}) \tag{95}$$
$$Q_{17}^{(Y)} = (\bar{\chi}\tau^a\chi)(\bar{Q}D\tau^a H + \text{h.c.}) \qquad Q_{18}^{(Y)} = (\bar{\chi}\mathrm{i}\gamma_5\tau^a\chi)(\bar{Q}D\tau^a H + \text{h.c.}) \tag{96}$$
$$Q_{19}^{(Y)} = (\bar{\chi}\tau^a\chi)(\mathrm{i}\bar{Q}D\tau^a H + \text{h.c.}) \qquad Q_{20}^{(Y)} = (\bar{\chi}\mathrm{i}\gamma_5\tau^a\chi)(\mathrm{i}\bar{Q}D\tau^a H + \text{h.c.}) \tag{97}$$
$$Q_{21}^{(Y)} = (\bar{\chi}\sigma_{\mu\nu}\chi)(\bar{Q}\sigma^{\mu\nu}DH + \text{h.c.}) \qquad Q_{22}^{(Y)} = (\bar{\chi}\sigma_{\mu\nu}\chi)(\mathrm{i}\bar{Q}\sigma^{\mu\nu}DH + \text{h.c.}) \tag{98}$$
$$Q_{23}^{(Y)} = (\bar{\chi}\sigma_{\mu\nu}\tau^a\chi)(\bar{Q}\sigma^{\mu\nu}D\tau^a H + \text{h.c.}) \qquad Q_{24}^{(Y)} = (\bar{\chi}\sigma_{\mu\nu}\tau^a\chi)(\mathrm{i}\bar{Q}\sigma^{\mu\nu}D\tau^a H + \text{h.c.}) \tag{99}$$
$$Q_{25}^{(Y)} = (\bar{\chi}\chi)(\bar{Q}U\epsilon H + \text{h.c.}) \qquad Q_{26}^{(Y)} = (\bar{\chi}\mathrm{i}\gamma_5\chi)(\bar{Q}U\epsilon H + \text{h.c.}) \tag{100}$$
$$Q_{27}^{(Y)} = (\bar{\chi}\chi)(\mathrm{i}\bar{Q}U\epsilon H + \text{h.c.}) \qquad Q_{28}^{(Y)} = (\bar{\chi}\mathrm{i}\gamma_5\chi)(\mathrm{i}\bar{Q}U\epsilon H + \text{h.c.}) \tag{101}$$
$$Q_{29}^{(Y)} = (\bar{\chi}\tau^a\chi)(\bar{Q}U\tau^a\epsilon H + \text{h.c.}) \qquad Q_{30}^{(Y)} = (\bar{\chi}\mathrm{i}\gamma_5\tau^a\chi)(\bar{Q}U\tau^a\epsilon H + \text{h.c.}) \tag{102}$$
$$Q_{31}^{(Y)} = (\bar{\chi}\tau^a\chi)(\mathrm{i}\bar{Q}U\tau^a\epsilon H + \text{h.c.}) \qquad Q_{32}^{(Y)} = (\bar{\chi}\mathrm{i}\gamma_5\tau^a\chi)(\mathrm{i}\bar{Q}U\tau^a\epsilon H + \text{h.c.}) \tag{103}$$
$$Q_{33}^{(Y)} = (\bar{\chi}\sigma_{\mu\nu}\chi)(\bar{Q}\sigma^{\mu\nu}U\epsilon H + \text{h.c.}) \qquad Q_{34}^{(Y)} = (\bar{\chi}\sigma_{\mu\nu}\chi)(\mathrm{i}\bar{Q}\sigma^{\mu\nu}U\epsilon H + \text{h.c.}) \tag{104}$$
$$Q_{35}^{(Y)} = (\bar{\chi}\sigma_{\mu\nu}\tau^a\chi)(\bar{Q}\sigma^{\mu\nu}U\tau^a\epsilon H + \text{h.c.}) \qquad Q_{36}^{(Y)} = (\bar{\chi}\sigma_{\mu\nu}\tau^a\chi)(\mathrm{i}\bar{Q}\sigma^{\mu\nu}U\tau^a\epsilon H + \text{h.c.}) \tag{105}$$

Four-Higgs operators

$$Q_1^{(4H)} = (\bar{\chi}\chi)\ H^\dagger H\ H^\dagger H \qquad Q_2^{(4H)} = (\bar{\chi}\mathrm{i}\gamma_5\chi)\ H^\dagger H\ H^\dagger H \tag{106}$$
$$Q_3^{(4H)} = (\bar{\chi}\tau^a\chi)\ H^\dagger\tau^a H\ H^\dagger H \qquad Q_4^{(4H)} = (\bar{\chi}\mathrm{i}\gamma_5\tau^a\chi)\ H^\dagger\tau^a H\ H^\dagger H \tag{107}$$

Four-Fermion operators

$$Q_1^{(d2F)} = (\bar{\chi}\sigma^{\mu\nu}\chi)\partial_\mu(\bar{E}\gamma_\nu E) \qquad Q_2^{(d2F)} = (\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\chi)\partial_\mu(\bar{E}\gamma_\nu E) \tag{108}$$
$$Q_3^{(d2F)} = (\bar{\chi}\sigma^{\mu\nu}\chi)\partial_\mu(\bar{D}\gamma_\nu D) \qquad Q_4^{(d2F)} = (\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\chi)\partial_\mu(\bar{D}\gamma_\nu D) \tag{109}$$
$$Q_5^{(d2F)} = (\bar{\chi}\sigma^{\mu\nu}\chi)\partial_\mu(\bar{U}\gamma_\nu U) \qquad Q_6^{(d2F)} = (\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\chi)\partial_\mu(\bar{U}\gamma_\nu U) \tag{110}$$
$$Q_7^{(d2F)} = (\bar{\chi}\sigma^{\mu\nu}\chi)\partial_\mu(\bar{L}\gamma_\nu L) \qquad Q_8^{(d2F)} = (\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\chi)\partial_\mu(\bar{L}\gamma_\nu L) \tag{111}$$
$$Q_9^{(d2F)} = (\bar{\chi}\sigma^{\mu\nu}\tau^a\chi)\partial_\mu(\bar{L}\tau^a\gamma_\nu L) \qquad Q_{10}^{(d2F)} = (\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\tau^a\chi)\partial_\mu(\bar{L}\tau^a\gamma_\nu L) \tag{112}$$
$$Q_{11}^{(d2F)} = (\bar{\chi}\sigma^{\mu\nu}\chi)\partial_\mu(\bar{Q}\gamma_\nu Q) \qquad Q_{12}^{(d2F)} = (\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\chi)\partial_\mu(\bar{Q}\gamma_\nu Q) \tag{113}$$
$$Q_{13}^{(d2F)} = (\bar{\chi}\sigma^{\mu\nu}\tau^a\chi)\partial_\mu(\bar{Q}\tau^a\gamma_\nu Q) \qquad Q_{14}^{(d2F)} = (\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\tau^a\chi)\partial_\mu(\bar{Q}\tau^a\gamma_\nu Q) \tag{114}$$

Two-Higgs operators

$$Q_1^{(2d2H)} = (\bar{\chi}\chi)D_\mu H^\dagger D^\mu H \qquad Q_2^{(2d2H)} = (\bar{\chi}\mathrm{i}\gamma_5\chi)D_\mu H^\dagger D^\mu H \tag{115}$$
$$Q_3^{(2d2H)} = (\bar{\chi}\tau^a\chi)D_\mu H^\dagger\tau^a D^\mu H \qquad Q_4^{(2d2H)} = (\bar{\chi}\mathrm{i}\gamma_5\tau^a\chi)D_\mu H^\dagger\tau^a D^\mu H \tag{116}$$
$$Q_5^{(2d2H)} = \mathrm{i}(\bar{\chi}\sigma^{\mu\nu}\chi)D_\mu H^\dagger D_\nu H \qquad Q_6^{(2d2H)} = \mathrm{i}(\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\chi)D_\mu H^\dagger D_\nu H \tag{117}$$
$$Q_7^{(2d2H)} = \mathrm{i}(\bar{\chi}\sigma^{\mu\nu}\tau^a\chi)D_\mu H^\dagger\tau^a D_\nu H \qquad Q_8^{(2d2H)} = \mathrm{i}(\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\tau^a\chi)D_\mu H^\dagger\tau^a D_\nu H \tag{118}$$

5.5 The full basis below the electroweak scale

Below the electroweak scale, the different field content and broken symmetry lead to a new full basis of effective operators. The Higgs boson, massive gauge bosons and top quark are integrated out. Note that operators below the electroweak scale and their Wilson coefficients are always denoted with calligraphic letters. In the rest of this section, the field f can be chosen from $\{u, d, c, s, b, e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau\}$. However, for neutrino currents we keep only those operators where the Dirac indices are contracted via a single γ_μ due to their chirality. The possible hypercharge-conserving operator classes up to mass dimension seven are given by

- $\mathcal{Q}^{(G)} \propto \bar{\chi}\chi\, G$ - Gauge (Dim 5)
- $\mathcal{Q}^{(2F)} \propto \bar{\chi}\chi\, \bar{f}f$ - Four-Fermion (Dim 6)
- $\mathcal{Q}^{(d2F)} \propto D\, \bar{\chi}\chi\, \bar{f}f$ - Four-Fermion (Dim 7)
- $\mathcal{Q}^{(GG)} \propto \bar{\chi}\chi\, GG$ - Gauge-Gauge (Dim 7)

For the Majorana case, we again include an additional factor 1/2 for every dark matter bilinear.

Gauge operators (Dim 5)

$$\mathcal{Q}_1^{(G)} = (\bar{\chi}\sigma^{\mu\nu}\chi)F_{\mu\nu} \qquad \mathcal{Q}_2^{(G)} = (\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\chi)F_{\mu\nu} \tag{119}$$

Four-Fermion operators (Dim 6)

$$\mathcal{Q}_{1,f}^{(2F)} = (\bar{\chi}\chi)\,(\bar{f}f) \qquad \mathcal{Q}_{2,f}^{(2F)} = (\bar{\chi}\mathrm{i}\gamma_5\chi)\,(\bar{f}f) \tag{120}$$
$$\mathcal{Q}_{3,f}^{(2F)} = (\bar{\chi}\chi)\,(\bar{f}\mathrm{i}\gamma_5 f) \qquad \mathcal{Q}_{4,f}^{(2F)} = (\bar{\chi}\mathrm{i}\gamma_5\chi)\,(\bar{f}\mathrm{i}\gamma_5 f) \tag{121}$$
$$\mathcal{Q}_{5,f}^{(2F)} = (\bar{\chi}\gamma^\mu\chi)\,(\bar{f}\gamma_\mu f) \qquad \mathcal{Q}_{6,f}^{(2F)} = (\bar{\chi}\gamma^\mu\gamma_5\chi)\,(\bar{f}\gamma_\mu f) \tag{122}$$
$$\mathcal{Q}_{7,f}^{(2F)} = (\bar{\chi}\gamma^\mu\chi)\,(\bar{f}\gamma_\mu\gamma_5 f) \qquad \mathcal{Q}_{8,f}^{(2F)} = (\bar{\chi}\gamma^\mu\gamma_5\chi)\,(\bar{f}\gamma_\mu\gamma_5 f) \tag{123}$$
$$\mathcal{Q}_{9,f}^{(2F)} = (\bar{\chi}\sigma^{\mu\nu}\chi)\,(\bar{f}\sigma_{\mu\nu} f) \qquad \mathcal{Q}_{10,f}^{(2F)} = (\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\chi)\,(\bar{f}\sigma_{\mu\nu} f) \tag{124}$$

Four-Fermion operators (Dim 7)

$$\mathcal{Q}_{1,f}^{(d2F)} = (\bar{\chi}\gamma^\mu\chi)\,(\bar{f}\;\mathrm{i}\overleftrightarrow{D}_\mu\;f) \qquad \mathcal{Q}_{2,f}^{(d2F)} = (\bar{\chi}\gamma^\mu\chi)\,(\bar{f}\mathrm{i}\gamma_5\;\mathrm{i}\overleftrightarrow{D}_\mu\;f) \tag{125}$$
$$\mathcal{Q}_{3,f}^{(d2F)} = (\bar{\chi}\gamma^\mu\gamma_5\chi)\,(\bar{f}\;\mathrm{i}\overleftrightarrow{D}_\mu\;f) \qquad \mathcal{Q}_{4,f}^{(d2F)} = (\bar{\chi}\gamma^\mu\gamma_5\chi)\,(\bar{f}\mathrm{i}\gamma_5\;\mathrm{i}\overleftrightarrow{D}_\mu\;f) \tag{126}$$
$$\mathcal{Q}_{5,f}^{(d2F)} = (\bar{\chi}\sigma^{\mu\nu}\chi)\,\partial_\mu(\bar{f}\gamma_\nu f) \qquad \mathcal{Q}_{6,f}^{(d2F)} = (\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\chi)\,\partial_\mu(\bar{f}\gamma_\nu f) \tag{127}$$
$$\mathcal{Q}_{7,f}^{(d2F)} = (\bar{\chi}\sigma^{\mu\nu}\chi)\,\partial_\mu(\bar{f}\gamma_\nu\gamma_5 f) \qquad \mathcal{Q}_{8,f}^{(d2F)} = (\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\chi)\,\partial_\mu(\bar{f}\gamma_\nu\gamma_5 f) \tag{128}$$

Gauge-Gauge operators (Dim 7)

$$\mathcal{Q}_{1}^{(GG)} = (\bar{\chi}\chi)\,F^{\mu\nu}F_{\mu\nu} \qquad \mathcal{Q}_{2}^{(GG)} = (\bar{\chi}\mathrm{i}\gamma_5\chi)\,F^{\mu\nu}F_{\mu\nu} \tag{129}$$
$$\mathcal{Q}_{3}^{(GG)} = (\bar{\chi}\chi)\,F^{\mu\nu}\tilde{F}_{\mu\nu} \qquad \mathcal{Q}_{4}^{(GG)} = (\bar{\chi}\mathrm{i}\gamma_5\chi)\,F^{\mu\nu}\tilde{F}_{\mu\nu} \tag{130}$$
$$\mathcal{Q}_{5}^{(GG)} = (\bar{\chi}\chi)\,G^{a\mu\nu}G^a_{\mu\nu} \qquad \mathcal{Q}_{6}^{(GG)} = (\bar{\chi}\mathrm{i}\gamma_5\chi)\,G^{a\mu\nu}G^a_{\mu\nu} \tag{131}$$
$$\mathcal{Q}_{7}^{(GG)} = (\bar{\chi}\chi)\,G^{a\mu\nu}\tilde{G}^a_{\mu\nu} \qquad \mathcal{Q}_{8}^{(GG)} = (\bar{\chi}\mathrm{i}\gamma_5\chi)\,G^{a\mu\nu}\tilde{G}^a_{\mu\nu} \tag{132}$$

6 Matching to the broken SM

In this chapter, we discuss the matching of the full UV basis to a basis below the electroweak scale. We perform the matching at tree-level, additionally cutting off all operators with a mass dimension higher than seven. While our DM model suggests a vanishing hypercharge, we perform the calculation for a general $Y_\chi \neq 0$ as a reference. While we will only include the top Yukawa coupling for the RG running of the full operator basis, we keep all Yukawa couplings here due to their phenomenological relevance in the low energy limit, as it is also done in [1].

The first section describes a chiral rotation of the dark matter field, which is necessary for a canonical mass term, as well as the effects of such a redefinition on the UV Wilson coefficients. In the remaining two sections of this chapter, we provide the matching relations of UV operators onto a basis below the electroweak scale, for the cases of both light and electroweak-scale dark matter, respectively.

6.1 Chiral rotation of the dark matter field

As previously stated, we perform a chiral rotation of the χ field after breaking the electroweak symmetry, analogously to the treatment in [1] and discussed in more detail in [44]. We implement a field redefinition

$$\chi' = \exp\left(\frac{\mathrm{i}}{2}\gamma_5\phi\right)\chi = \cos\left(\frac{\phi}{2}\right)\chi + \sin\left(\frac{\phi}{2}\right)\mathrm{i}\gamma_5\chi \tag{133}$$

This is necessary, since the effective UV operators with four Higgs fields can induce an imaginary mass term $\bar{\chi}\mathrm{i}\gamma_5\chi$ after electroweak symmetry breaking. This term vanishes after the redefinition by choosing ϕ according to

$$\tan\phi = \frac{C_2^{(H4)} + \frac{1}{2}Y_\chi C_4^{(H4)}}{C_1^{(H4)} + \frac{1}{2}Y_\chi C_3^{(H4)} - \frac{4\Lambda^3}{v_{EW}^4}m_\chi}, \tag{134}$$

where C are the Wilson coefficients of the appropriate UV operators. We choose the solution ϕ that gives rise to a positive mass, which takes the form

$$m'_\chi = m_\chi\cos\phi - \frac{v_{EW}^4}{4\Lambda^3}\left(C_1^{(H4)}\cos\phi + \frac{1}{2}Y_\chi C_3^{(H4)}\cos\phi + C_2^{(H4)}\sin\phi + \frac{1}{2}Y_\chi C_4^{(H4)}\sin\phi\right). \tag{135}$$

The chiral rotation leads to a mixing of Wilson coefficients when switching to the operator basis where χ is replaced with χ'. Their mixing follows from the following relations:

$$
\begin{aligned}
\alpha\bar{\chi}\chi + \beta\bar{\chi}\mathrm{i}\gamma_5\chi &= (\alpha\cos\phi + \beta\sin\phi)\,\bar{\chi}'\chi' + (\beta\cos\phi - \alpha\sin\phi)\,\bar{\chi}'\mathrm{i}\gamma_5\chi' \\
\alpha\bar{\chi}\sigma^{\mu\nu}\chi + \beta\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\chi &= (\alpha\cos\phi + \beta\sin\phi)\,\bar{\chi}'\sigma^{\mu\nu}\chi' + (\beta\cos\phi - \alpha\sin\phi)\,\bar{\chi}'\sigma^{\mu\nu}\mathrm{i}\gamma_5\chi'
\end{aligned}
\tag{136}
$$

where α and β stand for Wilson coefficients. (Axial-)Vector currents are unaffected. Our basis above the electroweak scale is arranged in such a way that this translates to

$$
\begin{pmatrix} C_i^{(X)\prime} \\ C_{i+1}^{(X)\prime} \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} C_i^{(X)} \\ C_{i+1}^{(X)} \end{pmatrix} \tag{137}
$$

$$
= \begin{pmatrix} 1 & -x(C_2^{(H4)} + \frac{1}{2}Y_\chi C_4^{(H4)}) \\ x(C_2^{(H4)} + \frac{1}{2}Y_\chi C_4^{(H4)}) & 1 \end{pmatrix} \begin{pmatrix} C_i^{(X)} \\ C_{i+1}^{(X)} \end{pmatrix}, \tag{138}
$$

where X is any operator category and i any odd integer. The second equality, which we supply for consistent power-counting for the case of electroweak-scale dark matter, follows by expanding in $x = \frac{v_{EW}^4}{4\Lambda^3 m_\chi}$, where no ambiguity in sign arises since Eq. (135) implies that the positive-mass solution of ϕ is close to zero in this scenario.

Note that in the case of some Yukawa-like operators, the mixing takes place between operators that do not contain any $\mathrm{i}\gamma_5$, which follows by using Eq. (53) to shift the $\mathrm{i}\gamma_5$ to the SM current and exploiting the SM field's chirality.

In the following treatment, we assume that the χ redefinition was already performed and drop all primes on χ, m_χ and Wilson coefficients.

6.2 Light DM

Using our basis, we write down the effective Lagrangian for the light dark matter case as

$$
\mathcal{L}_{\text{eff}} = \sum_{X,a} \hat{\mathcal{C}}_a^{(X)}\ \mathcal{Q}_a^{(X)}, \tag{139}
$$

where X is any operator class. We follow [1] in our notation, so that we factor out the electroweak scale and dark matter masses by defining

$$
\hat{\mathcal{C}}_a^{(X)} = \sum_{n,m} \frac{\mathcal{C}_a^{(X)\{n,m\}}}{\Lambda^{d-n-m-4} v_{EW}^n m_\chi^m} \tag{140}
$$

for the Wilson coefficients below the electroweak scale, where d is the mass dimension of the operator $\mathcal{Q}_a^{(X)}$.

The only contribution resulting from gauge interactions is given by Z boson exchange, yielding

$$
\mathcal{C}_{5,u_i}^{(2F)\{2,0\}} = \frac{3 - 8s_w^2}{3} Y_\chi, \tag{141}
$$

$$
\mathcal{C}_{7,u_i}^{(2F)\{2,0\}} = -Y_\chi, \tag{142}
$$

$$\mathcal{C}_{5,d_i}^{(2F)\{2,0\}} = -\frac{3-4s_w^2}{3}Y_\chi, \tag{143}$$
$$\mathcal{C}_{7,d_i}^{(2F)\{2,0\}} = +Y_\chi, \tag{144}$$
$$\mathcal{C}_{5,e_i}^{(2F)\{2,0\}} = -(1-4s_w^2)Y_\chi, \tag{145}$$
$$\mathcal{C}_{7,e_i}^{(2F)\{2,0\}} = +Y_\chi. \tag{146}$$

The contributions from **Gauge-Gauge** UV operators to those below the electroweak scale result from performing the substitution (29). The coefficients for operators with a photon field strength tensor are given by

$$\mathcal{C}_1^{(GG)\{0,0\}} = c_w^2 C_1^{(GG)} - Y_\chi s_w c_w C_5^{(GG)} + s_w^2 C_{15}^{(GG)}, \tag{147}$$
$$\mathcal{C}_2^{(GG)\{0,0\}} = c_w^2 C_2^{(GG)} - Y_\chi s_w c_w C_6^{(GG)} + s_w^2 C_{16}^{(GG)}, \tag{148}$$
$$\mathcal{C}_3^{(GG)\{0,0\}} = c_w^2 C_3^{(GG)} - Y_\chi s_w c_w C_7^{(GG)} + s_w^2 C_{17}^{(GG)}, \tag{149}$$
$$\mathcal{C}_4^{(GG)\{0,0\}} = c_w^2 C_4^{(GG)} - Y_\chi s_w c_w C_8^{(GG)} + s_w^2 C_{18}^{(GG)}. \tag{150}$$

The operators with gluon tensors match exactly:

$$\mathcal{C}_5^{(GG)\{0,0\}} = C_{11}^{(GG)}, \tag{151}$$
$$\mathcal{C}_6^{(GG)\{0,0\}} = C_{12}^{(GG)}, \tag{152}$$
$$\mathcal{C}_7^{(GG)\{0,0\}} = C_{13}^{(GG)}, \tag{153}$$
$$\mathcal{C}_8^{(GG)\{0,0\}} = C_{14}^{(GG)}. \tag{154}$$

At tree level, there are no contributions from $C_9^{(GG)}, C_{10}^{(GG)}, C_{19}^{(GG)}$ and $C_{20}^{(GG)}$.

The **Gauge-Higgs** operators contribute to the dimension-five dipole operators by replacing the Higgs with its vacuum expectation value. The Wilson coefficients are given by

$$\mathcal{C}_1^{(G)\{-2,0\}} = \frac{c_w}{2}C_1^{(GH)} + \frac{Y_\chi c_w}{4}C_3^{(GH)} - \frac{s_w}{4}C_5^{(GH)} - \frac{Y_\chi s_w}{2}C_7^{(GH)}, \tag{155}$$
$$\mathcal{C}_2^{(G)\{-2,0\}} = \frac{c_w}{2}C_2^{(GH)} + \frac{Y_\chi c_w}{4}C_4^{(GH)} - \frac{s_w}{4}C_6^{(GH)} - \frac{Y_\chi s_w}{2}C_8^{(GH)}. \tag{156}$$

At tree level, there are no contributions from $C_9^{(GH)}$ and $C_{10}^{(GH)}$.

The **Yukawa-like** operators match onto dimension-six Four-Fermion operators by replacing the Higgs with its vacuum expectation value. Additionally, the **Four-Higgs** operators contribute to the χ mass, as discussed in section 6.1, and also the dimension-six Four-Fermion operators after integrating out one Higgs and replacing the others with its vacuum expectation value. For up-type quarks ($u_1 = u, u_2 = c$) this yields the coefficients

$$\mathcal{C}_{1,u_i}^{(2F)\{-1,0\}} = \frac{1}{\sqrt{2}}C_{25,i}^{(Y)} + \frac{Y_\chi}{2\sqrt{2}}C_{29,i}^{(Y)} - \frac{y_{u_i}}{\sqrt{2}\lambda}\left(C_1^{(4H)} + \frac{Y_\chi}{2}C_3^{(4H)}\right), \tag{157}$$
$$\mathcal{C}_{2,u_i}^{(2F)\{-1,0\}} = \frac{1}{\sqrt{2}}C_{26,i}^{(Y)} + \frac{Y_\chi}{2\sqrt{2}}C_{30,i}^{(Y)} - \frac{y_{u_i}}{\sqrt{2}\lambda}\left(C_2^{(4H)} + \frac{Y_\chi}{2}C_4^{(4H)}\right), \tag{158}$$

$$\mathcal{C}^{(2F)\{-1,0\}}_{3,u_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{27,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{31,i}, \tag{159}$$

$$\mathcal{C}^{(2F)\{-1,0\}}_{4,u_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{28,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{32,i}, \tag{160}$$

$$\mathcal{C}^{(2F)\{-1,0\}}_{9,u_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{33,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{35,i}, \tag{161}$$

$$\mathcal{C}^{(2F)\{-1,0\}}_{10,u_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{34,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{36,i}. \tag{162}$$

for down-type quarks ($d_1 = d, d_2 = s, d_3 = b$)

$$\mathcal{C}^{(2F)\{-1,0\}}_{1,d_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{13,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{17,i} - \frac{y_{d_i}}{\sqrt{2}\lambda} \left(C^{(4H)}_1 + \frac{Y_\chi}{2} C^{(4H)}_3 \right), \tag{163}$$

$$\mathcal{C}^{(2F)\{-1,0\}}_{2,d_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{14,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{18,i} - \frac{y_{d_i}}{\sqrt{2}\lambda} \left(C^{(4H)}_2 + \frac{Y_\chi}{2} C^{(4H)}_4 \right), \tag{164}$$

$$\mathcal{C}^{(2F)\{-1,0\}}_{3,d_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{15,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{19,i}, \tag{165}$$

$$\mathcal{C}^{(2F)\{-1,0\}}_{4,d_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{16,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{20,i}, \tag{166}$$

$$\mathcal{C}^{(2F)\{-1,0\}}_{9,d_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{21,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{23,i}, \tag{167}$$

$$\mathcal{C}^{(2F)\{-1,0\}}_{10,d_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{22,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{24,i}, \tag{168}$$

and for charged leptons ($e_1 = e, e_2 = \mu, e_3 = \tau$)

$$\mathcal{C}^{(2F)\{-1,0\}}_{1,e_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{1,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{5,i} - \frac{y_{e_i}}{\sqrt{2}\lambda} \left(C^{(4H)}_1 + \frac{Y_\chi}{2} C^{(4H)}_3 \right), \tag{169}$$

$$\mathcal{C}^{(2F)\{-1,0\}}_{2,e_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{2,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{6,i} - \frac{y_{e_i}}{\sqrt{2}\lambda} \left(C^{(4H)}_2 + \frac{Y_\chi}{2} C^{(4H)}_4 \right), \tag{170}$$

$$\mathcal{C}^{(2F)\{-1,0\}}_{3,e_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{3,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{7,i}, \tag{171}$$

$$\mathcal{C}^{(2F)\{-1,0\}}_{4,e_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{4,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{8,i}, \tag{172}$$

$$\mathcal{C}^{(2F)\{-1,0\}}_{9,e_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{9,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{11,i}, \tag{173}$$

$$\mathcal{C}^{(2F)\{-1,0\}}_{10,e_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{10,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{12,i}. \tag{174}$$

The **Four-Fermion** UV operators directly match onto the dimension-seven Four-Fermion operators after symmetry breaking. For up-type quarks the coefficients below the electroweak scale are given by

$$\mathcal{C}^{(d2F)\{0,0\}}_{5,u_i} = \frac{1}{2} C^{(d2F)}_{5,i} + \frac{1}{2} C^{(d2F)}_{11,i} - \frac{Y_\chi}{4} C^{(d2F)}_{13,i}, \tag{175}$$

$$\mathcal{C}^{(d2F)\{0,0\}}_{6,u_i} = \frac{1}{2} C^{(d2F)}_{6,i} + \frac{1}{2} C^{(d2F)}_{12,i} - \frac{Y_\chi}{4} C^{(d2F)}_{14,i}, \tag{176}$$

$$\mathcal{C}^{(d2F)\{0,0\}}_{7,u_i} = \frac{1}{2}C^{(d2F)}_{5,i} - \frac{1}{2}C^{(d2F)}_{11,i} + \frac{Y_\chi}{4}C^{(d2F)}_{13,i}, \tag{177}$$
$$\mathcal{C}^{(d2F)\{0,0\}}_{8,u_i} = \frac{1}{2}C^{(d2F)}_{6,i} - \frac{1}{2}C^{(d2F)}_{12,i} + \frac{Y_\chi}{4}C^{(d2F)}_{14,i}, \tag{178}$$

for down-type quarks

$$\mathcal{C}^{(d2F)\{0,0\}}_{5,d_i} = \frac{1}{2}C^{(d2F)}_{3,i} + \frac{1}{2}C^{(d2F)}_{11,i} + \frac{Y_\chi}{4}C^{(d2F)}_{13,i}, \tag{179}$$
$$\mathcal{C}^{(d2F)\{0,0\}}_{6,d_i} = \frac{1}{2}C^{(d2F)}_{4,i} + \frac{1}{2}C^{(d2F)}_{12,i} + \frac{Y_\chi}{4}C^{(d2F)}_{14,i}, \tag{180}$$
$$\mathcal{C}^{(d2F)\{0,0\}}_{7,d_i} = \frac{1}{2}C^{(d2F)}_{3,i} - \frac{1}{2}C^{(d2F)}_{11,i} - \frac{Y_\chi}{4}C^{(d2F)}_{13,i}, \tag{181}$$
$$\mathcal{C}^{(d2F)\{0,0\}}_{8,d_i} = \frac{1}{2}C^{(d2F)}_{4,i} - \frac{1}{2}C^{(d2F)}_{12,i} - \frac{Y_\chi}{4}C^{(d2F)}_{14,i}, \tag{182}$$

for charged leptons

$$\mathcal{C}^{(d2F)\{0,0\}}_{5,e_i} = \frac{1}{2}C^{(d2F)}_{1,i} + \frac{1}{2}C^{(d2F)}_{7,i} + \frac{Y_\chi}{4}C^{(d2F)}_{9,i}, \tag{183}$$
$$\mathcal{C}^{(d2F)\{0,0\}}_{6,e_i} = \frac{1}{2}C^{(d2F)}_{2,i} + \frac{1}{2}C^{(d2F)}_{8,i} + \frac{Y_\chi}{4}C^{(d2F)}_{10,i}, \tag{184}$$
$$\mathcal{C}^{(d2F)\{0,0\}}_{7,e_i} = \frac{1}{2}C^{(d2F)}_{1,i} - \frac{1}{2}C^{(d2F)}_{7,i} - \frac{Y_\chi}{4}C^{(d2F)}_{9,i}, \tag{185}$$
$$\mathcal{C}^{(d2F)\{0,0\}}_{8,e_i} = \frac{1}{2}C^{(d2F)}_{2,i} - \frac{1}{2}C^{(d2F)}_{8,i} - \frac{Y_\chi}{4}C^{(d2F)}_{10,i}, \tag{186}$$

and for neutrinos ($\nu_1 = \nu_e, \nu_2 = \nu_\mu, \nu_3 = \nu_\tau$)

$$\mathcal{C}^{(d2F)\{0,0\}}_{5,\nu_i} = C^{(d2F)}_{7,i} - \frac{Y_\chi}{2}C^{(d2F)}_{9,i}, \tag{187}$$
$$\mathcal{C}^{(d2F)\{0,0\}}_{6,\nu_i} = C^{(d2F)}_{8,i} - \frac{Y_\chi}{2}C^{(d2F)}_{10,i}. \tag{188}$$

The **Two-Higgs** operators in the UV do not contribute at tree level.

6.3 Electroweak scale DM

Since we did not consider loop contributions in the last section, the results would also hold when not neglecting m_χ. Therefore, we match the UV basis to the HDMET by starting out with the coefficients of the last section and subsequently integrating out the small-component field of χ, as discussed in chapter 4. Equation (41) implies the following useful replacement formulae that we employ for tree-level matching for the Dirac case:

$$\bar\chi\chi \to \bar\chi_v\chi_v + \mathcal{O}(1/m_\chi^2), \tag{189}$$
$$\bar\chi i\gamma_5\chi \to \frac{1}{m_\chi}\partial_\mu(\bar\chi_v S^\mu\chi_v) + \mathcal{O}(1/m_\chi^2), \tag{190}$$
$$\bar\chi\gamma^\mu\chi \to \bar\chi_v v^\mu\chi_v - \frac{1}{2m_\chi}\bar\chi_v\, i\overleftrightarrow{\partial}^\mu\,\chi_v + \frac{1}{m_\chi}\partial_\nu(\bar\chi_v S_\rho v_\eta\chi_v)\varepsilon^{\mu\nu\rho\eta} + \mathcal{O}(1/m_\chi^2), \tag{191}$$

$$\bar{\chi}\gamma^\mu\gamma_5\chi \to 2\bar{\chi}_v S^\mu \chi_v + \frac{1}{m_\chi}\bar{\chi}_v (S\cdot \overleftrightarrow{\mathrm{i}\partial}) v^\mu \chi_v + \mathcal{O}(1/m_\chi^2), \tag{192}$$

$$\begin{aligned}\bar{\chi}\sigma^{\mu\nu}\chi \to{}& 2\bar{\chi}_v S_\rho v_\eta \chi_v \varepsilon^{\mu\nu\rho\eta} - \frac{1}{m_\chi}\bar{\chi}_v S_\sigma \overleftrightarrow{\mathrm{i}\partial}_\rho\, \chi_v \varepsilon^{\sigma\rho\mu\nu} + \frac{1}{m_\chi}\partial^{[\mu}(\bar{\chi}_v v^{\nu]}\chi_v) \\ &+ \frac{1}{4m_\chi^2}\,\bar{\chi}_v \overleftarrow{\not{\partial}}_\perp \sigma_\perp^{\mu\nu} \not{\partial}_\perp \chi_v + \mathcal{O}(1/m_\chi^3),\end{aligned} \tag{193}$$

$$\begin{aligned}\bar{\chi}\sigma^{\mu\nu}\mathrm{i}\gamma_5\chi \to{}& 4\bar{\chi}_v S^{[\mu} v^{\nu]}\chi_v + \frac{2}{m_\chi}\bar{\chi}_v S^{[\mu}\, \overleftrightarrow{\mathrm{i}\partial}^{\nu]}\, \chi_v - \frac{1}{2m_\chi}\partial_\alpha(\bar{\chi}_v v_\beta \chi_v)\varepsilon^{\mu\nu\alpha\beta} \\ &- \frac{1}{8m_\chi^2}\,\bar{\chi}_v \overleftarrow{\not{\partial}}_\perp \sigma^\perp_{\alpha\beta} \not{\partial}_\perp \chi_v\, \varepsilon^{\alpha\beta\mu\nu} + \mathcal{O}(1/m_\chi^3),\end{aligned} \tag{194}$$

where we have used the dark matter spin operator $S^\mu = \gamma_\perp^\mu\gamma_5/2$. We need $1/m_\chi^2$ terms for tensor currents since they already appear at dimension five in the dipole operators. Indices in (square) brackets are (anti-)symmetrized, where our convention is to divide by the number of permutations. For the first formula, we additionally used the equation of motion. These formulae were checked to agree with [1] up to different choices of conventions.

For the Majorana case, these relations come with an additional factor 2 on the right hand side due to the modified tree-level relation. This factor cancels with the customary factor 1/2 in the definition of our operators, so that the matching conditions look exactly like the Dirac case.

With the replacement rules in place, we construct a basis that spans all operators that are induced by our UV theory at tree level. We did not use our program to construct a full basis due to the different treatment of equations of motions this would require. Analogously to the previous case, we write

$$\mathcal{L}_{\rm eff} = \sum_{d,a} \hat{\tilde{\mathcal{C}}}_a^{(d)}\ \tilde{\mathcal{Q}}_a^{(d)}, \tag{195}$$

where d is the mass dimension of the operator $\tilde{\mathcal{Q}}_a^{(d)}$, and again factor out mass scales from Wilson coefficients with

$$\hat{\tilde{\mathcal{C}}}_a^{(d)} = \sum_{n,m} \frac{\tilde{\mathcal{C}}_a^{(d)\{n,m\}}}{\Lambda^{d-n-m-4} v_{EW}^n m_\chi^m}, \tag{196}$$

where the tilde is used to distinguish these quantities from the light dark matter scenario. Operators with gauge bosons are given by

$$\tilde{\mathcal{Q}}_1^{(G)} = (\bar{\chi}_v S_\mu v_\nu \chi_v)\, F^{\mu\nu}, \qquad \tilde{\mathcal{Q}}_2^{(G)} = (\bar{\chi}_v S_\mu v_\nu \chi_v)\, \tilde{F}^{\mu\nu}, \tag{197}$$

$$\tilde{\mathcal{Q}}_1^{(dG)} = (\bar{\chi}_v S_\mu\, \overleftrightarrow{\mathrm{i}\partial}_\nu\, \chi_v)\, F^{\mu\nu}, \qquad \tilde{\mathcal{Q}}_2^{(dG)} = (\bar{\chi}_v S_\mu\, \overleftrightarrow{\mathrm{i}\partial}_\nu\, \chi_v)\, \tilde{F}^{\mu\nu}, \tag{198}$$

$$\tilde{\mathcal{Q}}_1^{(2dG)} = (\bar{\chi}_v \overleftarrow{\not{\partial}}_\perp \sigma_\perp^{\mu\nu} \not{\partial}_\perp \chi_v)\, F^{\mu\nu}, \qquad \tilde{\mathcal{Q}}_2^{(2dG)} = (\bar{\chi}_v \overleftarrow{\not{\partial}}_\perp \sigma_\perp^{\mu\nu} \not{\partial}_\perp \chi_v)\, \tilde{F}^{\mu\nu}, \tag{199}$$

$$\tilde{\mathcal{Q}}_1^{(GG)} = (\bar{\chi}_v\chi_v)\, F^{\mu\nu}F_{\mu\nu}, \qquad \tilde{\mathcal{Q}}_2^{(GG)} = (\bar{\chi}_v\chi_v)\, F^{\mu\nu}\tilde{F}_{\mu\nu}, \tag{200}$$

$$\tilde{\mathcal{Q}}_3^{(GG)} = (\bar{\chi}_v\chi_v)\, G^{a\mu\nu}G^a_{\mu\nu}, \qquad \tilde{\mathcal{Q}}_4^{(GG)} = (\bar{\chi}_v\chi_v)\, G^{a\mu\nu}\tilde{G}^a_{\mu\nu}. \tag{201}$$

The four-fermion operators are given by

$$\tilde{\mathcal{Q}}_{1,f}^{(2F)} = (\bar{\chi}_v\chi_v)\,(\bar{f}f), \qquad \tilde{\mathcal{Q}}_{2,f}^{(2F)} = (\bar{\chi}_v\chi_v)\,(\bar{f}\mathrm{i}\gamma_5 f), \tag{202}$$

$$
\begin{aligned}
\tilde{\mathcal{Q}}_{3,f}^{(2F)} &= (\bar{\chi}_v\chi_v)\,(\bar{f}\not{v}f), & \tilde{\mathcal{Q}}_{4,f}^{(2F)} &= (\bar{\chi}_v S^\mu\chi_v)\,(\bar{f}\gamma_\mu f), && (203)\\
\tilde{\mathcal{Q}}_{5,f}^{(2F)} &= (\bar{\chi}_v\chi_v)\,(\bar{f}\not{v}\gamma_5 f), & \tilde{\mathcal{Q}}_{6,f}^{(2F)} &= (\bar{\chi}_v S^\mu\chi_v)\,(\bar{f}\gamma_\mu\gamma_5 f), && (204)\\
\tilde{\mathcal{Q}}_{7,f}^{(2F)} &= (\bar{\chi}_v S^\mu v^\nu\chi_v)\,(\bar{f}\sigma_{\mu\nu}f), & \tilde{\mathcal{Q}}_{8,f}^{(2F)} &= (\bar{\chi}_v S_\mu v_\nu\chi_v)\,(\bar{f}\sigma_{\rho\eta}f)\varepsilon^{\mu\nu\rho\eta}, && (205)\\
\tilde{\mathcal{Q}}_{1,f}^{(d2F)} &= \partial_\mu(\bar{\chi}_v S^\mu\chi_v)\,(\bar{f}f), & \tilde{\mathcal{Q}}_{2,f}^{(d2F)} &= \partial_\mu(\bar{\chi}_v S^\mu\chi_v)\,(\bar{f}\mathrm{i}\gamma_5 f), && (206)\\
\tilde{\mathcal{Q}}_{3,f}^{(d2F)} &= (\bar{\chi}_v\ \mathrm{i}\overleftrightarrow{\partial}^\mu\ \chi_v)\,(\bar{f}\gamma_\mu f), & \tilde{\mathcal{Q}}_{4,f}^{(d2F)} &= (\bar{\chi}_v\ \mathrm{i}\overleftrightarrow{\partial}^\mu\ \chi_v)\,(\bar{f}\gamma_\mu\gamma_5 f), && (207)\\
\tilde{\mathcal{Q}}_{5,f}^{(d2F)} &= \partial_\nu(\bar{\chi}_v S_\rho v_\eta\chi_v)\,(\bar{f}\gamma_\mu f)\varepsilon^{\mu\nu\rho\eta}, & \tilde{\mathcal{Q}}_{6,f}^{(d2F)} &= \partial_\nu(\bar{\chi}_v S_\rho v_\eta\chi_v)\,(\bar{f}\gamma_\mu\gamma_5 f)\varepsilon^{\mu\nu\rho\eta}, && (208)\\
\tilde{\mathcal{Q}}_{7,f}^{(d2F)} &= (\bar{\chi}_v S\cdot\ \mathrm{i}\overleftrightarrow{\partial}\ \chi_v)\,(\bar{f}\not{v}f), & \tilde{\mathcal{Q}}_{8,f}^{(d2F)} &= (\bar{\chi}_v S\cdot\ \mathrm{i}\overleftrightarrow{\partial}\ \chi_v)\,(\bar{f}\not{v}\gamma_5 f), && (209)\\
\tilde{\mathcal{Q}}_{9,f}^{(d2F)} &= (\bar{\chi}_v S^\mu\ \mathrm{i}\overleftrightarrow{\partial}^\nu\ \chi_v)\,(\bar{f}\sigma_{\mu\nu}f), & \tilde{\mathcal{Q}}_{10,f}^{(d2F)} &= \partial_\mu(\bar{\chi}_v v_\nu\chi_v)\,(\bar{f}\sigma_{\mu\nu}f), && (210)\\
\tilde{\mathcal{Q}}_{11,f}^{(d2F)} &= (\bar{\chi}_v S^\mu\ \mathrm{i}\overleftrightarrow{\partial}^\nu\ \chi_v)\,(\bar{f}\sigma_{\rho\eta}f)\varepsilon^{\mu\nu\rho\eta}, & \tilde{\mathcal{Q}}_{12,f}^{(d2F)} &= \partial_\mu(\bar{\chi}_v v_\nu\chi_v)\,(\bar{f}\sigma_{\rho\eta}f)\varepsilon^{\mu\nu\rho\eta}, && (211)\\
\tilde{\mathcal{Q}}_{13,f}^{(d2F)} &= (\bar{\chi}_v\chi_v)\,(\bar{f}v\cdot\ \mathrm{i}\overleftrightarrow{D}\ f) & \tilde{\mathcal{Q}}_{14,f}^{(d2F)} &= (\bar{\chi}_v\chi_v)\,(\bar{f}\mathrm{i}\gamma_5 v\cdot\ \mathrm{i}\overleftrightarrow{D}\ f) && (212)\\
\tilde{\mathcal{Q}}_{15,f}^{(d2F)} &= (\bar{\chi}_v S^\mu\chi_v)\,(\bar{f}\ \mathrm{i}\overleftrightarrow{D}_\mu\ f) & \tilde{\mathcal{Q}}_{16,f}^{(d2F)} &= (\bar{\chi}_v S^\mu\chi_v)\,(\bar{f}\mathrm{i}\gamma_5\ \mathrm{i}\overleftrightarrow{D}_\mu\ f) && (213)\\
\tilde{\mathcal{Q}}_{17,f}^{(d2F)} &= (\bar{\chi}_v S^\mu v^\nu\chi_v)\,\partial_\mu(\bar{f}\gamma_\nu f), & \tilde{\mathcal{Q}}_{18,f}^{(d2F)} &= (\bar{\chi}_v S_\mu v_\nu\chi_v)\,\partial_\rho(\bar{f}\gamma_\eta f)\varepsilon^{\mu\nu\rho\eta}, && (214)\\
\tilde{\mathcal{Q}}_{19,f}^{(d2F)} &= (\bar{\chi}_v S^\mu v^\nu\chi_v)\,\partial_\mu(\bar{f}\gamma_\nu\gamma_5 f), & \tilde{\mathcal{Q}}_{20,f}^{(d2F)} &= (\bar{\chi}_v S_\mu v_\nu\chi_v)\,\partial_\rho(\bar{f}\gamma_\eta\gamma_5 f)\varepsilon^{\mu\nu\rho\eta}, && (215)
\end{aligned}
$$

where f is any fermion, again keeping for neutrino currents only those operators with a γ_μ.

In an intermediate step, we perform the tree-level matching of our full basis below the electroweak scale for a general dark matter mass to this basis and then used the results of the last section to subsequently perform the matching with the full UV operator basis, yielding the following contributions via the operator class with one gauge boson:

$$
\begin{aligned}
\tilde{\mathcal{C}}_1^{(G)\{-2,0\}} &= 2c_w C_2^{(GH)} + Y_\chi c_w C_4^{(GH)} - s_w C_6^{(GH)} - 2Y_\chi s_w C_8^{(GH)}, && (216)\\
\tilde{\mathcal{C}}_2^{(G)\{-2,0\}} &= 2c_w C_1^{(GH)} + Y_\chi c_w C_3^{(GH)} - s_w C_5^{(GH)} - 2Y_\chi s_w C_7^{(GH)}, && (217)\\
\tilde{\mathcal{C}}_1^{(dG)\{-2,1\}} &= c_w C_2^{(GH)} + \frac{Y_\chi c_w}{2} C_4^{(GH)} - \frac{s_w}{2} C_6^{(GH)} - Y_\chi s_w C_8^{(GH)} && (218)\\
\tilde{\mathcal{C}}_2^{(dG)\{-2,1\}} &= -c_w C_1^{(GH)} - \frac{Y_\chi c_w}{2} C_3^{(GH)} + \frac{s_w}{2} C_5^{(GH)} + Y_\chi s_w C_7^{(GH)}, && (219)\\
\tilde{\mathcal{C}}_1^{(2dG)\{-2,2\}} &= \frac{c_w}{8} C_1^{(GH)} + \frac{Y_\chi c_w}{16} C_3^{(GH)} - \frac{s_w}{16} C_5^{(GH)} - \frac{Y_\chi s_w}{8} C_7^{(GH)}, && (220)\\
\tilde{\mathcal{C}}_2^{(2dG)\{-2,2\}} &= -\frac{c_w}{8} C_2^{(GH)} - \frac{Y_\chi c_w}{16} C_4^{(GH)} + \frac{s_w}{16} C_6^{(GH)} + \frac{Y_\chi s_w}{8} C_8^{(GH)}, && (221)\\
\tilde{\mathcal{C}}_{3,f}^{(2F)\{-2,1\}} &= eQ_f\left(\frac{c_w}{2} C_1^{(GH)} + \frac{Y_\chi c_w}{4} C_3^{(GH)} - \frac{s_w}{4} C_5^{(GH)} - \frac{Y_\chi s_w}{2} C_7^{(GH)}\right), && (222)
\end{aligned}
$$

where the last, four-fermion contribution arises by using the equations of motion and f is any charged fermion. The Wilson coefficients for the four-fermion class with up-type quarks are given by

$$
\tilde{\mathcal{C}}_{1,u_i}^{(2F)\{-1,0\}} = \frac{1}{\sqrt{2}} C_{25,i}^{(Y)} + \frac{Y_\chi}{2\sqrt{2}} C_{29,i}^{(Y)} - \frac{y_{u_i}}{\sqrt{2}\lambda}\left(C_1^{(4H)} + \frac{Y_\chi}{2} C_3^{(4H)}\right), \qquad (223)
$$

$$\tilde{C}^{(2F)\{-1,0\}}_{2,u_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{27,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{31,i}, \tag{224}$$

$$\tilde{C}^{(2F)\{2,0\}}_{3,u_i} = \frac{3 - 8s_w^2}{3} Y_\chi, \tag{225}$$

$$\tilde{C}^{(2F)\{2,0\}}_{5,u_i} = -Y_\chi, \tag{226}$$

$$\tilde{C}^{(2F)\{-1,0\}}_{7,u_i} = 2\sqrt{2} C^{(Y)}_{34,i} + \sqrt{2} Y_\chi C^{(Y)}_{36,i}, \tag{227}$$

$$\tilde{C}^{(2F)\{-1,0\}}_{8,u_i} = \sqrt{2} C^{(Y)}_{33,i} + \frac{Y_\chi}{\sqrt{2}} C^{(Y)}_{35,i}, \tag{228}$$

$$\tilde{C}^{(d2F)\{-1,0\}}_{1,u_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{26,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{30,i} - \frac{y_{u_i}}{\sqrt{2}\lambda} \left(C^{(4H)}_2 + \frac{Y_\chi}{2} C^{(4H)}_4 \right), \tag{229}$$

$$\tilde{C}^{(d2F)\{-1,0\}}_{2,u_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{28,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{32,i}, \tag{230}$$

$$\tilde{C}^{(d2F)\{2,1\}}_{3,u_i} = -\frac{3 - 8s_w^2}{6} Y_\chi, \tag{231}$$

$$\tilde{C}^{(d2F)\{2,1\}}_{4,u_i} = \frac{Y_\chi}{2}, \tag{232}$$

$$\tilde{C}^{(d2F)\{2,1\}}_{5,u_i} = \frac{3 - 8s_w^2}{3} Y_\chi, \tag{233}$$

$$\tilde{C}^{(d2F)\{2,1\}}_{6,u_i} = -Y_\chi, \tag{234}$$

$$\tilde{C}^{(d2F)\{-1,1\}}_{9,u_i} = \sqrt{2} C^{(Y)}_{34,i} + \frac{Y_\chi}{\sqrt{2}} C^{(Y)}_{36,i}, \tag{235}$$

$$\tilde{C}^{(d2F)\{-1,1\}}_{10,u_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{33,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{35,i}, \tag{236}$$

$$\tilde{C}^{(d2F)\{-1,1\}}_{11,u_i} = -\frac{1}{\sqrt{2}} C^{(Y)}_{33,i} - \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{35,i}, \tag{237}$$

$$\tilde{C}^{(d2F)\{-1,1\}}_{12,u_i} = -\frac{1}{2\sqrt{2}} C^{(Y)}_{34,i} - \frac{Y_\chi}{4\sqrt{2}} C^{(Y)}_{36,i}, \tag{238}$$

$$\tilde{C}^{(d2F)\{0,0\}}_{17,u_i} = 2C^{(d2F)}_{6,i} + 2C^{(d2F)}_{12,i} - Y_\chi C^{(d2F)}_{14,i}, \tag{239}$$

$$\tilde{C}^{(d2F)\{0,0\}}_{18,u_i} = C^{(d2F)}_{5,i} + C^{(d2F)}_{11,i} - \frac{Y_\chi}{2} C^{(d2F)}_{13,i}, \tag{240}$$

$$\tilde{C}^{(d2F)\{0,0\}}_{19,u_i} = 2C^{(d2F)}_{6,i} - 2C^{(d2F)}_{12,i} + Y_\chi C^{(d2F)}_{14,i}, \tag{241}$$

$$\tilde{C}^{(d2F)\{0,0\}}_{20,u_i} = C^{(d2F)}_{5,i} - C^{(d2F)}_{11,i} + \frac{Y_\chi}{2} C^{(d2F)}_{13,i}, \tag{242}$$

while for down-type quarks we find

$$\tilde{C}^{(2F)\{-1,0\}}_{1,d_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{13,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{17,i} - \frac{y_{d_i}}{\sqrt{2}\lambda} \left(C^{(4H)}_1 + \frac{Y_\chi}{2} C^{(4H)}_3 \right), \tag{243}$$

$$\tilde{C}^{(2F)\{-1,0\}}_{2,d_i} = \frac{1}{\sqrt{2}} C^{(Y)}_{15,i} + \frac{Y_\chi}{2\sqrt{2}} C^{(Y)}_{19,i}, \tag{244}$$

$$\tilde{C}^{(2F)\{2,0\}}_{3,d_i} = -\frac{3 - 4s_w^2}{3} Y_\chi, \tag{245}$$

$$\tilde{C}^{(2F)\{2,0\}}_{5,d_i} = +Y_\chi, \tag{246}$$

$$\tilde{\mathcal{C}}_{7,d_i}^{(2F)\{-1,0\}} = 2\sqrt{2}C_{22,i}^{(Y)} + \sqrt{2}Y_\chi C_{24,i}^{(Y)}, \tag{247}$$

$$\tilde{\mathcal{C}}_{8,d_i}^{(2F)\{-1,0\}} = \sqrt{2}C_{21,i}^{(Y)} + \frac{Y_\chi}{\sqrt{2}}C_{23,i}^{(Y)}, \tag{248}$$

$$\tilde{\mathcal{C}}_{1,d_i}^{(d2F)\{-1,0\}} = \frac{1}{\sqrt{2}}C_{14,i}^{(Y)} + \frac{Y_\chi}{2\sqrt{2}}C_{18,i}^{(Y)} - \frac{y_{d_i}}{\sqrt{2}\lambda}\left(C_2^{(4H)} + \frac{Y_\chi}{2}C_4^{(4H)}\right), \tag{249}$$

$$\tilde{\mathcal{C}}_{2,d_i}^{(d2F)\{-1,0\}} = \frac{1}{\sqrt{2}}C_{16,i}^{(Y)} + \frac{Y_\chi}{2\sqrt{2}}C_{20,i}^{(Y)}, \tag{250}$$

$$\tilde{\mathcal{C}}_{3,d_i}^{(d2F)\{2,1\}} = \frac{3-4s_w^2}{6}Y_\chi, \tag{251}$$

$$\tilde{\mathcal{C}}_{4,d_i}^{(d2F)\{2,1\}} = -\frac{Y_\chi}{2}, \tag{252}$$

$$\tilde{\mathcal{C}}_{5,d_i}^{(d2F)\{2,1\}} = -\frac{3-4s_w^2}{3}Y_\chi, \tag{253}$$

$$\tilde{\mathcal{C}}_{6,d_i}^{(d2F)\{2,1\}} = Y_\chi, \tag{254}$$

$$\tilde{\mathcal{C}}_{9,d_i}^{(d2F)\{-1,1\}} = \sqrt{2}C_{22,i}^{(Y)} + \frac{Y_\chi}{\sqrt{2}}C_{24,i}^{(Y)}, \tag{255}$$

$$\tilde{\mathcal{C}}_{10,d_i}^{(d2F)\{-1,1\}} = \frac{1}{\sqrt{2}}C_{21,i}^{(Y)} + \frac{Y_\chi}{2\sqrt{2}}C_{23,i}^{(Y)}, \tag{256}$$

$$\tilde{\mathcal{C}}_{11,d_i}^{(d2F)\{-1,1\}} = -\frac{1}{\sqrt{2}}C_{21,i}^{(Y)} - \frac{Y_\chi}{2\sqrt{2}}C_{23,i}^{(Y)}, \tag{257}$$

$$\tilde{\mathcal{C}}_{12,d_i}^{(d2F)\{-1,1\}} = -\frac{1}{2\sqrt{2}}C_{22,i}^{(Y)} - \frac{Y_\chi}{4\sqrt{2}}C_{24,i}^{(Y)}, \tag{258}$$

$$\tilde{\mathcal{C}}_{17,d_i}^{(d2F)\{0,0\}} = 2C_{4,i}^{(d2F)} + 2C_{12,i}^{(d2F)} + Y_\chi C_{14,i}^{(d2F)}, \tag{259}$$

$$\tilde{\mathcal{C}}_{18,d_i}^{(d2F)\{0,0\}} = C_{3,i}^{(d2F)} + C_{11,i}^{(d2F)} + \frac{Y_\chi}{2}C_{13,i}^{(d2F)}, \tag{260}$$

$$\tilde{\mathcal{C}}_{19,d_i}^{(d2F)\{0,0\}} = 2C_{4,i}^{(d2F)} - 2C_{12,i}^{(d2F)} - Y_\chi C_{14,i}^{(d2F)}, \tag{261}$$

$$\tilde{\mathcal{C}}_{20,d_i}^{(d2F)\{0,0\}} = C_{3,i}^{(d2F)} - C_{11,i}^{(d2F)} - \frac{Y_\chi}{2}C_{13,i}^{(d2F)}. \tag{262}$$

For charged leptons, we find the contributions

$$\tilde{\mathcal{C}}_{1,e_i}^{(2F)\{-1,0\}} = \frac{1}{\sqrt{2}}C_{1,i}^{(Y)} + \frac{Y_\chi}{2\sqrt{2}}C_{5,i}^{(Y)} - \frac{y_{e_i}}{\sqrt{2}\lambda}\left(C_1^{(4H)} + \frac{Y_\chi}{2}C_3^{(4H)}\right), \tag{263}$$

$$\tilde{\mathcal{C}}_{2,e_i}^{(2F)\{-1,0\}} = \frac{1}{\sqrt{2}}C_{3,i}^{(Y)} + \frac{Y_\chi}{2\sqrt{2}}C_{7,i}^{(Y)}, \tag{264}$$

$$\tilde{\mathcal{C}}_{3,e_i}^{(2F)\{2,0\}} = -(1-4s_w^2)Y_\chi, \tag{265}$$

$$\tilde{\mathcal{C}}_{5,e_i}^{(2F)\{2,0\}} = +Y_\chi, \tag{266}$$

$$\tilde{\mathcal{C}}_{7,e_i}^{(2F)\{-1,0\}} = 2\sqrt{2}C_{10,i}^{(Y)} + \sqrt{2}Y_\chi C_{12,i}^{(Y)}, \tag{267}$$

$$\tilde{\mathcal{C}}_{8,e_i}^{(2F)\{-1,0\}} = \sqrt{2}C_{9,i}^{(Y)} + \frac{Y_\chi}{\sqrt{2}}C_{11,i}^{(Y)}, \tag{268}$$

$$\tilde{\mathcal{C}}_{1,e_i}^{(d2F)\{-1,0\}} = \frac{1}{\sqrt{2}}C_{2,i}^{(Y)} + \frac{Y_\chi}{2\sqrt{2}}C_{6,i}^{(Y)} - \frac{y_{e_i}}{\sqrt{2}\lambda}\left(C_2^{(4H)} + \frac{Y_\chi}{2}C_4^{(4H)}\right), \tag{269}$$

$$\tilde{\mathcal{C}}^{(d2F)\{-1,0\}}_{2,e_i} = \frac{1}{\sqrt{2}}C^{(Y)}_{4,i} + \frac{Y_\chi}{2\sqrt{2}}C^{(Y)}_{8,i}, \tag{270}$$
$$\tilde{\mathcal{C}}^{(d2F)\{2,1\}}_{3,e_i} = \frac{1-4s_w^2}{2}Y_\chi, \tag{271}$$
$$\tilde{\mathcal{C}}^{(d2F)\{2,1\}}_{4,e_i} = -\frac{Y_\chi}{2} \tag{272}$$
$$\tilde{\mathcal{C}}^{(d2F)\{2,1\}}_{5,e_i} = -(1-4s_w^2)Y_\chi, \tag{273}$$
$$\tilde{\mathcal{C}}^{(d2F)\{2,1\}}_{6,e_i} = Y_\chi \tag{274}$$
$$\tilde{\mathcal{C}}^{(d2F)\{-1,1\}}_{9,e_i} = \sqrt{2}C^{(Y)}_{10,i} + \frac{Y_\chi}{\sqrt{2}}C^{(Y)}_{12,i}, \tag{275}$$
$$\tilde{\mathcal{C}}^{(d2F)\{-1,1\}}_{10,e_i} = \frac{1}{\sqrt{2}}C^{(Y)}_{9,i} + \frac{Y_\chi}{2\sqrt{2}}C^{(Y)}_{11,i}, \tag{276}$$
$$\tilde{\mathcal{C}}^{(d2F)\{-1,1\}}_{11,e_i} = -\frac{1}{\sqrt{2}}C^{(Y)}_{9,i} - \frac{Y_\chi}{2\sqrt{2}}C^{(Y)}_{11,i}, \tag{277}$$
$$\tilde{\mathcal{C}}^{(d2F)\{-1,1\}}_{12,e_i} = -\frac{1}{2\sqrt{2}}C^{(Y)}_{10,i} - \frac{Y_\chi}{4\sqrt{2}}C^{(Y)}_{12,i}, \tag{278}$$
$$\tilde{\mathcal{C}}^{(d2F)\{0,0\}}_{17,e_i} = 2C^{(d2F)}_{2,i} + 2C^{(d2F)}_{8,i} + Y_\chi C^{(d2F)}_{10,i}, \tag{279}$$
$$\tilde{\mathcal{C}}^{(d2F)\{0,0\}}_{18,e_i} = C^{(d2F)}_{1,i} + C^{(d2F)}_{7,i} + \frac{Y_\chi}{2}C^{(d2F)}_{9,i}, \tag{280}$$
$$\tilde{\mathcal{C}}^{(d2F)\{0,0\}}_{19,e_i} = 2C^{(d2F)}_{2,i} - 2C^{(d2F)}_{8,i} - Y_\chi C^{(d2F)}_{10,i}, \tag{281}$$
$$\tilde{\mathcal{C}}^{(d2F)\{0,0\}}_{20,e_i} = C^{(d2F)}_{1,i} - C^{(d2F)}_{7,i} - \frac{Y_\chi}{2}C^{(d2F)}_{9,i}, \tag{282}$$

and for neutrinos

$$\tilde{\mathcal{C}}^{(d2F)\{0,0\}}_{17,\nu_i} = 4C^{(d2F)}_{8,i} - 2Y_\chi C^{(d2F)}_{10,i}, \tag{283}$$
$$\tilde{\mathcal{C}}^{(d2F)\{0,0\}}_{18,\nu_i} = 2C^{(d2F)}_{7,i} - Y_\chi C^{(d2F)}_{9,i}. \tag{284}$$

Finally, the contributions to the Gauge-Gauge class are given by

$$\tilde{\mathcal{C}}^{(GG)\{0,0\}}_{1} = c_w^2 C^{(GG)}_{1} - Y_\chi s_w c_w C^{(GG)}_{5} + s_w^2 C^{(GG)}_{15}, \tag{285}$$
$$\tilde{\mathcal{C}}^{(GG)\{0,0\}}_{2} = c_w^2 C^{(GG)}_{3} - Y_\chi s_w c_w C^{(GG)}_{7} + s_w^2 C^{(GG)}_{17}, \tag{286}$$
$$\tilde{\mathcal{C}}^{(GG)\{0,0\}}_{3} = C^{(GG)}_{11}, \tag{287}$$
$$\tilde{\mathcal{C}}^{(GG)\{0,0\}}_{4} = C^{(GG)}_{13}. \tag{288}$$

7 Preparation for the calculation of anomalous dimension matrices

In this chapter, we will discuss our preparation for the calculation of the running of UV Wilson coefficients under the renormalization group. To this end, we start by describing the background field method and renormalization of the effective theory at high energies. In the second section, we give an overview of the software used in our calculations. The last two sections provide more details on the simplification of algebraic expressions and integration of loop momenta, respectively.

7.1 Computation in a general background field gauge

For the abelian $U(1)$, we perform our calculation using a general R_ξ gauge. For the non-abelian symmetries, on the other hand, we use the background field gauge to simplify the renormalization procedure. This gauge, which was first introduced by DeWitt [45], first splits up the gauge field into a separate classical field and quantum field

$$W^a_{\mu\nu} \mapsto W^a_{\mu\nu} + \mathcal{W}^a_{\mu\nu} \tag{289}$$

$$G^a_{\mu\nu} \mapsto G^a_{\mu\nu} + \mathcal{G}^a_{\mu\nu} \tag{290}$$

where we represent the quantum fields with calligraphic letters. The classical field is treated as a fixed background, while the quantum field configurations are integrated over in the functional integral. Using the Fadeev-Popov method, we utilize a gauge-fixing condition which is covariant under the gauge symmetry associated with the background field. This functional determinant generates a Fadeev-Popov Lagrangian $\mathcal{L}^{\text{FP}}$, introducing the usual ghosts c_G and c_W. As opposed to the usual procedure, the classical field now still retains a local gauge symmetry. For a more in-depth introduction, see [46].

As a reference, the Lagrangian that this procedure generates and which was used for our computation is given by

$$\mathcal{L} = \mathcal{L}^{\text{gauge}} + \mathcal{L}^{\text{higgs}} + \mathcal{L}^{\text{matter}} + \mathcal{L}^{\text{yukawa}} + \mathcal{L}^{\text{FP}} + \mathcal{L}^{\text{DM}} + \mathcal{L}^{\text{eff}}$$

$$\mathcal{L}^{\text{gauge}} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}W^{\mu\nu}_a W^a_{\mu\nu} - \frac{1}{4}G^{\mu\nu}_a G^a_{\mu\nu}$$

$$\text{where} \begin{cases} B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \\ W^a_{\mu\nu} = \partial_\mu \left(W^a_\nu + \mathcal{W}^a_\nu\right) - \partial_\nu \left(W^a_\mu + \mathcal{W}^a_\mu\right) + g_2 \epsilon^{abc} \left(W^b_\mu + \mathcal{W}^b_\mu\right)\left(W^c_\nu + \mathcal{W}^c_\nu\right) \\ G^a_{\mu\nu} = \partial_\mu \left(G^a_\nu + \mathcal{G}^a_\nu\right) - \partial_\nu \left(G^a_\mu + \mathcal{G}^a_\mu\right) + g_3 f^{abc} \left(G^b_\mu + \mathcal{G}^b_\mu\right)\left(G^c_\nu + \mathcal{G}^c_\nu\right) \end{cases}$$

$$
\begin{aligned}
\mathcal{L}^{\mathrm{FP}} = &-\frac{1}{2\xi_1}\left(\partial^\mu B_\mu\right)^2 - \frac{1}{2\xi_2}\left(\partial^\mu \mathcal{W}^a_\mu + g_2\epsilon^{abc}W^b_\mu \mathcal{W}^{\mu c}\right)^2 - \frac{1}{2\xi_3}\left(\partial^\mu \mathcal{G}^a_\mu + g_3 f^{abc}G^b_\mu \mathcal{G}^{\mu c}\right)^2 \\
&- \bar{c}^a_W\left[\left(\partial_\mu\delta^{ac} + g_2\epsilon^{abc}W^b_\mu\right)\left(\partial^\mu\delta^{ce} + g_2\epsilon^{cde}W^{\mu d}\right) + g_2\left(\partial_\mu\delta^{ac} + g_2\epsilon^{abc}W^b_\mu\right)\epsilon^{cde}\mathcal{W}^{\mu d}\right]c^e_W \\
&- \bar{c}^a_G\left[\left(\partial_\mu\delta^{ac} + g_3 f^{abc}G^b_\mu\right)\left(\partial^\mu\delta^{ce} + g_3 f^{cde}G^{\mu d}\right) + g_3\left(\partial_\mu\delta^{ac} + g_3 f^{abc}G^b_\mu\right)f^{cde}\mathcal{G}^{\mu d}\right]c^e_G
\end{aligned}
$$

Here, the Higgs, Yukawa, (dark) matter and effective Lagrangians take the same form as in chapter 4, with the difference that the covariant derivative now takes the form

$$
D_\mu := \partial_\mu - \mathrm{i}g_1 B_\mu Y - \mathrm{i}g_2\left(W^a_\mu + \mathcal{W}^a_\mu\right)\tau^a - \mathrm{i}g_3\left(G^a_\mu + \mathcal{G}^b_\mu\right)t^b. \tag{291}
$$

We explicitly verified consistency of the generated Feynman rules with those supplied in [46]. As a consistency check of our calculations, we do not fix the gauge parameters ξ_i to a particular value.

In order to regularize the infinities arising through momentum integrals, we use 't Hoofts and Veltmans dimensional regularization [47]. For every field in our theory, we introduce formally infinite renormalization constants

$$
\phi^0 = Z_\phi^{1/2}\phi, \tag{292}
$$

relating the unrenormalized field ϕ^0 to the renormalized ϕ. This introduces new counter-term vertices that can be used to absorb any infinities that could arise in physical observables order-by-order in perturbation theory. We usually express Z_ϕ with

$$
\delta_\phi = Z_\phi - 1, \tag{293}
$$

which we set to vanish at constant order in the coupling constants. Using the methods that will be outlined in the rest of this section, we computed the field renormalization constants for all particles at one-loop level. They are given by

$$
\delta_B = \frac{\alpha_1}{4\pi\epsilon}\left(\frac{1}{6} - N_g - \frac{11}{27}N_g N_c - \frac{4}{3}Y_\chi^2 N_\chi^{SU(2)}\right), \tag{294}
$$

$$
\delta_W = \frac{\alpha_2}{4\pi\epsilon}\left(\frac{43}{6} - \frac{N_g(N_c+1)}{3} - \frac{4}{9}\mathcal{J}_\chi N_\chi^{SU(2)}\right), \tag{295}
$$

$$
\delta_G = \frac{\alpha_3}{4\pi\epsilon}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right), \tag{296}
$$

$$
\delta_H = \frac{\alpha_1}{4\pi\epsilon}\left(\frac{3}{4} - \frac{1}{4}\xi_1\right) + \frac{\alpha_2}{4\pi\epsilon}\left(\frac{9}{4} - \frac{3}{4}\xi_2\right) - \frac{y_t^2}{16\pi^2\epsilon}N_c, \tag{297}
$$

$$
\delta_Q = -\frac{\alpha_1}{4\pi\epsilon}\frac{1}{36}\xi_1 - \frac{\alpha_2}{4\pi\epsilon}\frac{3}{4}\xi_2 - \frac{\alpha_3}{4\pi\epsilon}\frac{4}{3}\xi_3 - \frac{y_t^2}{16\pi^2\epsilon}\frac{1}{2}, \tag{298}
$$

$$
\delta_U = -\frac{\alpha_1}{4\pi\epsilon}\frac{4}{9}\xi_1 - \frac{\alpha_3}{4\pi\epsilon}\frac{4}{3}\xi_3 - \frac{y_t^2}{16\pi^2\epsilon}, \tag{299}
$$

$$
\delta_D = -\frac{\alpha_1}{4\pi\epsilon}\frac{1}{9}\xi_1 - \frac{\alpha_3}{4\pi\epsilon}\frac{4}{3}\xi_3, \tag{300}
$$

$$\delta_L = -\frac{\alpha_1}{4\pi\epsilon}\frac{1}{4}\xi_1 - \frac{\alpha_2}{4\pi\epsilon}\frac{3}{4}\xi_2, \tag{301}$$

$$\delta_E = -\frac{\alpha_1}{4\pi\epsilon}\xi_1, \tag{302}$$

$$\delta_\chi = -\frac{\alpha_1}{4\pi\epsilon}Y_\chi^2\xi_1 - \frac{\alpha_2}{4\pi\epsilon}\mathcal{J}_\chi\xi_2, \tag{303}$$

where we define $\alpha_i := g_i^2/4\pi$, $\mathcal{J}_\chi = J_\chi(J_\chi+1)$ and keep the number of generations N_g, colours N_c and quark flavours N_f explicit. The term with the Yukawa coupling in δ_Q and δ_U only applies to the third generation quarks.

We also renormalize bare couplings g_i^0 according to

$$g_i^0 = Z_{g_1}g_i, \tag{304}$$

where we set $\delta_{g_i} = Z_{g_i} - 1$ again.

This is where we profit from using the background field gauge: it is constructed in such a way to yield a very useful relation between the renormalization constants of gauge couplings and the respective gauge field, which we here denote by A_i:

$$Z_{g_i} = Z_{A_i}^{-1/2}. \tag{305}$$

For the $U(1)$ coupling and gauge boson, this identity is of course implied by the Ward identity.

We can now read off the gauge vertex renormalization from Eqs. (294) – (296):

$$\delta_{g_1} = -\frac{\alpha_1}{8\pi\epsilon}\left(\frac{1}{6} - N_g - \frac{11}{27}N_gN_c - \frac{4}{3}Y_\chi^2N_\chi^{SU(2)}\right), \tag{306}$$

$$\delta_{g_2} = -\frac{\alpha_2}{8\pi\epsilon}\left(\frac{43}{6} - \frac{N_g(N_c+1)}{3} - \frac{4}{9}\mathcal{J}_\chi N_\chi^{SU(2)}\right), \tag{307}$$

$$\delta_{g_3} = -\frac{\alpha_3}{8\pi\epsilon}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right). \tag{308}$$

7.2 Overview of the software toolchain

All one-loop calculations are performed using a combination of different software packages. QGRAF [48] was used to generate all Feynman diagrams contributing to a particular matrix element. This was interfaced with custom code for the FORM [49] algebra system. PYTHON code was used to tie these components together and automate their execution, largely reducing the task of calculating anomalous dimensions and renormalization constants to supplying model files with particle content and Feynman rules.

The Feynman rules were mostly derived per hand. As a preparation for the calculation of the full anomalous dimension matrices, we implemented the standard model with an added fermion in background field gauge for the FEYNRULES 2.0 Mathematica package [50]. We plan to facilitate the automatic derivation for effective operator rules with a custom export routine for FEYNRULES.

This setup was extended by Maximilian Reininghaus to support integration with propagators of heavy particles and successfully applied to dimension five and six running for fermionic dark matter with a mass substantially above the electroweak scale. Moreover, we have checked agreement of our dimension-five anomalous dimension matrices against those found in [1].

7.3 Algebraic simplifications

Evaluation of group weights

To calculate the group weights of Feynman diagrams we need to simplify expressions of $SU(2)$ and $SU(3)$ generators and Levi-Civita symbols. For the fundamental representation of these groups, we employ the well-known algorithm due to Cvitanovic [51]. Specifically, we can reexpress all Levi-Civita Symbols as traces in the fundamental representation through

$$f^{abc} = -2\mathrm{i}\ \mathrm{Tr}\left(\tau^a\tau^b\tau^c - \tau^c\tau^b\tau^a\right), \tag{309}$$

where τ^a and f^{abc} are the fundamental generator and structure constant of the given $SU(N)$, respectively. We then go on to use the Fierz identity

$$\tau^a_{ij}\tau^a_{kl} = \frac{1}{2}\delta_{il}\delta_{kj} - \frac{1}{2N}\delta_{ij}\delta_{kl} \tag{310}$$

to eliminate all contracted adjoint indices.

The only case where this is not sufficient are expressions in the unspecified irreducible representation of $SU(2)$ under which the dark matter field transforms. Here, we instead use an algorithm that simplifies recursively using mainly the following identities valid in any such representation:

$$[\tau^a, \tau^b] = \frac{\mathrm{i}}{2}\epsilon^{abc}\tau^c, \qquad \mathrm{Tr}(\tau^a\tau^b) = J_\chi(J_\chi+1)(2J_\chi+1)/3, \tag{311}$$

$$\tau^a\tau^a = J_\chi(J_\chi+1), \qquad \tau^a\tau^b\tau^a = (J_\chi(J_\chi+1)-1)\tau^b, \tag{312}$$

while also exploiting this relation in the fundamental representation for standard model currents:

$$\tau^a\tau^b = \frac{1}{4} + \frac{\mathrm{i}}{2}\epsilon^{abc}\tau^c. \tag{313}$$

Projection of Dirac structure

The Dirac structure of matrix elements is simplified by projecting Dirac matrices onto the covariant standard basis using the trace as scalar product, yielding for any matrix M in bispinor space

$$M = s1 + p\gamma_5 + v_\mu\gamma^\mu + a_\mu\gamma^\mu\gamma_5 + t_{\mu\nu}\sigma^{\mu\nu}, \tag{314}$$

where the coefficients are given by

$$s = \frac{1}{4}\,\mathrm{Tr}(M), \qquad p = \frac{1}{4}\,\mathrm{Tr}(\gamma_5 M), \tag{315}$$

$$v_\mu = \frac{1}{4}\,\mathrm{Tr}(\gamma_\mu M), \qquad a_\mu = -\frac{1}{4}\,\mathrm{Tr}(\gamma_\mu\gamma_5 M), \tag{316}$$

$$t_{\mu\nu} = -t_{\nu\mu} = \frac{1}{8}\,\mathrm{Tr}(\sigma_{\mu\nu} M). \tag{317}$$

The traces and resulting Lorentz structure are computed using FORM's built-in algorithms.

7.4 Loop momentum integration

Infrared rearrangement

For the calculation of renormalization constants and anomalous dimension matrices, we only require the divergent $1/\epsilon$ terms in our one-loop calculations. This allows us to considerably simplify the integrals that arise by recursively applying a technique known as infrared rearrangement. This was introduced and motivated using an exact decomposition of propagators by Chetyrkin et al. in [52]. This decomposition is given by

$$\frac{1}{(q+p)^2 - M^2} = \frac{1}{q^2 - m^2} + \frac{M^2 - p^2 - 2pq - m^2}{q^2 - m^2} \frac{1}{(q+p)^2 - M^2}, \tag{318}$$

where q is the loop momentum, p is a linear combination of external momenta, M is the particle's mass and m is a new mass parameter introduced to regulate infrared divergences. While this might seem like an inconsequential rearrangement, its merit lies in the fact that the second term on the right hand side, which is the only term whose denominator still contains particle masses and external momenta, has an overall degree of divergence reduced by one. By repeatedly applying this replacement to all propagators, we obtain, after a finite number of steps, an expression where all divergent parts of the integral contain only the simple propagator $1/(q^2 - m^2)$.

Master integral for simple denominators

After we have performed infrared rearrangement and discarded convergent terms, the remaining integrals with an odd number of loop momenta in the numerator vanish by symmetry. Therefore, we are left with integrals of the type

$$I_n^k := \int \frac{d^D q}{(2\pi)^D} \frac{q^{\mu_1} \ldots q^{\mu_k}}{(q^2 - m^2)^n}, \tag{319}$$

where k is even, performing our calculation in $D = 4 - 2\epsilon$ dimensions.

Firstly, we simplify the tensor structure by noting that the result must be symmetric in all indices. Therefore we can make the following substitution below the integral

$$q^{\mu_1} \ldots q^{\mu_k} \mapsto \alpha_k (q^2)^{k/2} \, \Delta^{\mu_1 \cdots \mu_k}, \tag{320}$$

where $\Delta^{\mu_1 \cdots \mu_k}$ is the totally symmetric tensor built out of products of metric tensors, normalized to a factor 1 in front of every independent term (matching the definition of the FORM function `dd_()`). The proportionality constant α can be determined by demanding equality when contracting both sides with metric tensors, yielding

$$\alpha_k^{-1} = \prod_{i=0}^{\frac{k}{2}-1} (D + 2i). \tag{321}$$

The remaining integral is evaluated in $(4-2\epsilon)$-dimensional spherical coordinates after performing a Wick rotation, yielding

$$\begin{aligned}\int \frac{d^D q}{(2\pi)^D}\, \frac{(q^2)^{k/2}}{(q^2-m^2)^n} = &\frac{\mathrm{i}(-1)^{k/2+2-n}}{(4\pi)^{D/2}}\, \frac{(m^2)^{k/2+D/2-n}}{\Gamma(D/2)} \\ &\times \frac{\Gamma(n+D/2-2)\,\Gamma(n-k/2-D/2)}{\Gamma(n)}.\end{aligned} \tag{322}$$

Putting this together, we find that the integral converges for $\epsilon \to 0$ exactly when $2n-k>4$. Such integrals are finite and can therefore be disregarded for our purposes. On the other hand, when $2n-k<4$, the divergent term gets multiplied by the artificial mass parameter m, which must vanish since the original amplitude is independent of m. Thus, we limit ourselves to the case $2n-k=4$.

Using the Laurent expansion

$$\Gamma(\epsilon - N) = \frac{(-1)^N}{N!}\left(\frac{1}{\epsilon} - \gamma_E + \sum_{i=1}^{N} \frac{1}{i} + \mathcal{O}(\epsilon)\right), \tag{323}$$

where $N \in \mathbb{N}$ and γ_E is the Euler-Mascheroni constant, our final result is then given by

$$I_n^{2n-4} = \frac{\mathrm{i}\Delta^{\mu_1 \cdots \mu_k}}{16\pi^2\, 2^{k/2}\,(n-1)!}\frac{1}{\epsilon} + \mathcal{O}(\epsilon^0), \tag{324}$$

which can easily be implemented as a FORM replacement rule.

8 Example models

To motivate the usefulness of our extension of the effective field theory treatment to dimension seven operators, we will now give a number of models that generate effective operators starting at this mass dimension. These models have not been studied in detail but are supposed to serve only as illustrative examples. We limit ourselves to UV completions with interactions of mass dimension four. Note that this does not guarantee traditional renormalizability for models A and C due to the propagators of the massive vector fields they introduce.

All models start out with the SM fields and interactions as well as the gauge interactions of the DM field χ. The first three models generate effective interactions between the dark and standard model sector only at loop order. To prevent tree-level contributions to dimension six DM-DM operators, which after mixing could have a comparably large effect as the dimension seven DM-SM operators we are considering here, we introduce a heavy copy χ' of the dark matter field. For a minimal treatment we set the masses of all new particles to the same mediator scale Λ. α, β, γ are parameters whenever they appear.

Model A: Two-Higgs operators

To generate Two-Higgs operators, we introduce a new scalar mediator φ as well as a Proca field N_μ together with the following interactions:

$$\mathcal{L} \supset \alpha \bar{\chi}' \chi \varphi + \beta \varphi N_\mu D^\mu H + \text{h.c.} \tag{325}$$

By assigning the hypercharge -1/2 to φ and χ', we prevent mixing between χ and χ'. The left diagram displayed in Fig. 5 yields the following contributions in $\overline{MS}$ to our complete basis:

$$C_1^{(2d2H)} = -\frac{1}{16\pi^2 \Lambda^3} \frac{\alpha^2 \beta^2}{6} \tag{326}$$

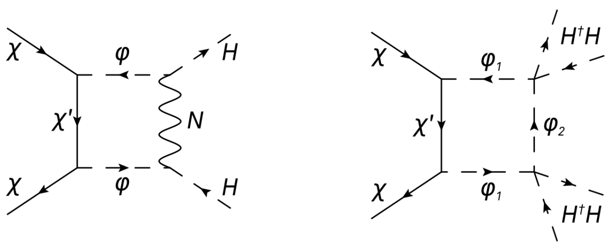


FIG. 5. Contribution to Two- and Four-Higgs operators in models A and B, respectively.

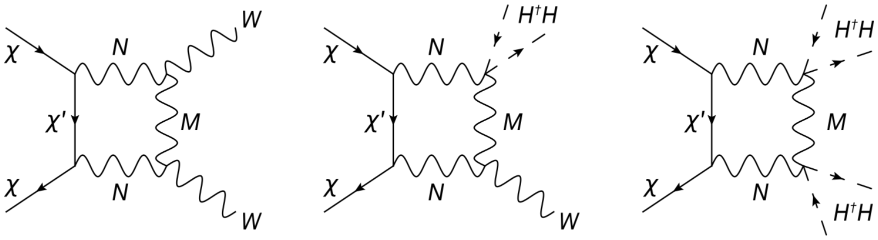


FIG. 6. Contribution to effective operators in model C.

The other Two-Higgs operators can be generated completely analogously by replacing the scalar DM interaction with a pseudoscalar, charging φ under $SU(2)$ and/or introducing a $\bar{\chi}\gamma_\mu\chi' N^\mu + \text{h.c.}$ interaction.

Model B: Four-Higgs operators

We introduce two scalars φ_1 and φ_2 with the interactions

$$\mathcal{L} \supset \alpha\bar{\chi}'\chi\varphi_1 + \beta\varphi_1^\dagger\varphi_2 H^\dagger H + \text{h.c.}, \tag{327}$$

where we can charge all heavy particles under an additional $U(1)$ to prevent mixing between χ and χ'. As per the right Feynman graph in Fig. 5, this gives the following contributions:

$$C_1^{(4H)} = \frac{1}{16\pi^2\Lambda^3}\frac{\alpha^2\beta^2}{6} \tag{328}$$

The remaining Four-Higgs operators could be generated with a pseudoscalar interaction and/or charging φ_2 under $SU(2)$.

Model C: Gauge-Gauge, Gauge-Higgs and Four-Higgs operators

In this model, we introduce two Proca fields N_μ and M_μ^a, where the latter transforms in the adjoint representation of one of the SM gauge symmetries. To illustrate, we choose $SU(2)$ and select the Lagrangian

$$\mathcal{L} \supset \alpha\bar{\chi}'\gamma_\mu\chi N^\mu + \beta M_a^\mu N^\nu W_{\mu\nu}^a + \gamma M_a^\mu N_\mu H^\dagger\tau^a H + \text{h.c.}, \tag{329}$$

where we can again prevent mixing between χ and χ' by assigning all new particles a charge under an additional $U(1)$. The diagrams in Fig. 6 yield contributions to the Gauge-Gauge, Gauge-Higgs and Four-Higgs operator classes:

$$C_{15}^{(GG)} = \frac{1}{16\pi^2\Lambda^3}\frac{\alpha^2\beta^2}{6} \tag{330}$$

$$C_5^{(GH)} = \frac{1}{16\pi^2\Lambda^3}\frac{\alpha^2\beta\gamma}{3} \tag{331}$$

$$C_1^{(4H)} = -\frac{1}{16\pi^2\Lambda^3}\frac{\alpha^2\gamma^2}{6} \tag{332}$$

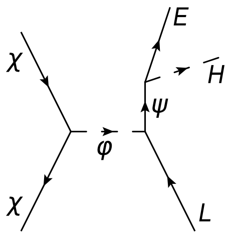


FIG. 7. Contribution to effective operators in model D.

Model D: Yukawa-like operators

We generate Yukawa-like operators by including a number of Yukawa couplings with a SM singlet scalar φ and a Dirac particle ψ, which we choose to transform under the standard model like a given quark or lepton doublet. For leptons, for example, we can introduce the interactions

$$\mathcal{L} \supset \alpha\bar{\chi}\chi\varphi + \beta\bar{L}\psi\varphi + \gamma\bar{\psi}EH + \text{h.c.}, \tag{333}$$

yielding a contribution

$$C_1^{(Y)} = \frac{\alpha\beta\gamma}{\Lambda^3}. \tag{334}$$

Choosing α or β complex as well as introducing a pseudoscalar interaction and/or charging φ under $SU(2)$ generates the remaining non-tensor Yukawa-like operators.

9 Summary

The effective field theory treatment of dark matter searches and particularly direct detection is very attractive for a model-independent interpretation and combination of experimental data. Moreover, it allows for the consistent study of renormalization group effects, which can considerably affect the phenomenology of UV theories and thereby their prospects of discovery.

In this thesis, we set out to expand the scope of an existing framework [1] to dimension seven UV operators. To this end, we provided a complete basis of dimension seven DM-SM operators invariant under the standard model symmetries and subsequently matched this basis onto effective theories valid below the electroweak scale, for both light and EW-scale dark matter. Moreover, we report on the assembly of a software framework for a largely automated calculation of anomalous dimensions in the UV regime.

In future work, we plan to interface FeynRules with our code as the last necessary step before calculating the anomalous dimensions. Additionally, we need to extend the low energy effective theories for our use. In particular, the additional tensor interactions must be included in Heavy Baryon Chiral Perturbation Theory. Finally, we would like to provide public code for automated RG running and matching to facilitate the use of our results.

Acknowledgements

I would like to express deep gratitude to my thesis advisor, Prof. Joachim Brod, from whom I have learnt a great deal and who was always available for incredibly interesting and helpful discussions.

I thank my best friend Thomas Jalili Tanha, my sister Luise, Jan Denkers and Stephan Escher for support and comments on the manuscript, as well as my parents Rüdiger and Beate for continuing support.

I am also grateful for the successful collaboration with Maximilian Reininghaus, Michele Tammaro and Jure Zupan.

References

[1] Fady Bishara et al. "Renormalization Group Effects in Dark Matter Interactions" (2017). In preparation for publication.

[2] Fady Bishara et al. "Chiral Effective Theory of Dark Matter Direct Detection" (2016). arXiv: `1611.00368 [hep-ph]`.

[3] Steven Weinberg. *The Quantum Theory of Fields. Vol. 2: Modern applications*. Cambridge University Press, 2013. ISBN: 9781139632478, 9780521670548, 9780521550024.

[4] Andrzej J. Buras. "Weak Hamiltonian, CP violation and rare decays". *Probing the standard model of particle interactions. Proceedings, Summer School in Theoretical Physics, NATO Advanced Study Institute, 68th session, Les Houches, France, July 28-September 5, 1997. Pt. 1, 2*. 1998, pp. 281–539. arXiv: `hep-ph/9806471 [hep-ph]`.

[5] H. Georgi. "Effective field theory". *Ann. Rev. Nucl. Part. Sci.* 43 (1993), pp. 209–252. DOI: `10.1146/annurev.ns.43.120193.001233`.

[6] Thomas Appelquist and J. Carazzone. "Infrared Singularities and Massive Fields". *Phys. Rev.* D11 (1975), p. 2856. DOI: `10.1103/PhysRevD.11.2856`.

[7] C. Patrignani et al. "Review of Particle Physics". *Chin. Phys.* C40.10 (2016), p. 100001. DOI: `10.1088/1674-1137/40/10/100001`.

[8] Gianfranco Bertone, Dan Hooper, and Joseph Silk. *Particle Dark Matter: Evidence, Candidates and Constraints*. Tech. rep. hep-ph/0404175. FERMILAB-Pub-2004-047-A. Batavia, IL: FERMILAB, 2004.

[9] F. Zwicky. "Die Rotverschiebung von extragalaktischen Nebeln". *Helv. Phys. Acta* 6 (1933). [Gen. Rel. Grav.41,207(2009)], pp. 110–127. DOI: `10.1007/s10714-008-0707-4`.

[10] Vera C. Rubin and W. Kent Ford Jr. "Rotation of the Andromeda Nebula from a Spectroscopic Survey of Emission Regions". *Astrophys. J.* 159 (1970), pp. 379–403. DOI: `10.1086/150317`.

[11] J I Read. *The Local Dark Matter Density*. Tech. rep. arXiv:1404.1938. JPHYSG-100038.R1. Comments: 62 pages, 11 Figures. To appear in Journal of Physics G. A high resolution version is available here: http://personal.ph.surrey.ac.uk/ jr0018/Papers/localdm_review.pdf. 2014.

[12] R. Adam et al. "Planck 2015 results. I. Overview of products and scientific results". *Astron. Astrophys.* 594 (2016), A1. DOI: `10.1051/0004-6361/201527101`. arXiv: `1502.01582 [astro-ph.CO]`.

[13] J. Tauber et al. "The Scientific programme of Planck" (2006). arXiv: `astro-ph/0604069 [astro-ph]`.

[14] Keith A Olive. *TASI Lectures on Dark Matter*. Tech. rep. astro-ph/0301505. UMN-TH-2127. TPI-MINN-2003-02. Minneapolis, MN: Minnesota Univ., 2003.

[15] R. D. Peccei and Helen R. Quinn. "CP Conservation in the Presence of Instantons". *Phys. Rev. Lett.* 38 (1977), pp. 1440–1443. DOI: `10.1103/PhysRevLett.38.1440`.

[16] Robin Bähre et al. *Any Light Particle Search II – Technical Design Report*. Aug. 2013. DOI: 10.1088/1748-0221/8/09/T09001. arXiv: 1302.5647v2 [physics.ins-det].

[17] M Milgrom. "A modification of the Newtonian dynamics as a possible alternative to the hidden mass". *Astrophys. J.* 270.2/1 (1983), pp. 365–370.

[18] Erik P. Verlinde. "Emergent Gravity and the Dark Universe" (2016). arXiv: 1611.02269 [hep-th].

[19] Fermi/LAT Collaboration, : and W. B. Atwood. "The Large Area Telescope on the Fermi Gamma-ray Space Telescope Mission". *Astrophys.J.* 697 (Feb. 2009), pp. 1071–1102. DOI: 10.1088/0004-637X/697/2/1071. arXiv: 0902.1089v1 [astro-ph.IM].

[20] The IceCube Collaboration et al. *The IceCube Neutrino Observatory - Contributions to ICRC 2015 Part IV: Searches for Dark Matter and Exotic Particles*. Nov. 2015. arXiv: 1510.05226v2 [astro-ph.HE].

[21] M. Aguilar et al. "First Result from the Alpha Magnetic Spectrometer on the International Space Station: Precision Measurement of the Positron Fraction in Primary Cosmic Rays of 0.5–350 GeV". *Phys. Rev. Lett.* 110 (2013), p. 141102. DOI: 10.1103/PhysRevLett.110.141102.

[22] Jennifer M Gaskins. *A review of indirect searches for particle dark matter*. Tech. rep. arXiv:1604.00014. Comments: 32 pages, 6 figures; invited review, accepted to Contemporary Physics. 2016.

[23] Paolo Panci. "New Directions in Direct Dark Matter Searches". *Adv. High Energy Phys.* 2014 (2014), p. 681312. DOI: 10.1155/2014/681312. arXiv: 1402.1507 [hep-ph].

[24] Andreas Crivellin, Francesco D'Eramo, and Massimiliano Procura. "New Constraints on Dark Matter Effective Theories from Standard Model Loops". *Phys. Rev. Lett.* 112 (May 2014), p. 191304. DOI: 10.1103/PhysRevLett.112.191304. arXiv: 1402.1173v2 [hep-ph].

[25] Marat Freytsis and Zoltan Ligeti. "On dark matter models with uniquely spin-dependent detection possibilities". *Phys. Rev.* D83 (2011), p. 115009. DOI: 10.1103/PhysRevD.83.115009. arXiv: 1012.5317 [hep-ph].

[26] Brian Feldstein, Masahiro Ibe, and Tsutomu T. Yanagida. "Hypercharged Dark Matter and Direct Detection as a Probe of Reheating". *Phys. Rev. Lett.* 112 (Oct. 2013), p. 101301. DOI: 10.1103/PhysRevLett.112.101301. arXiv: 1310.7495v1 [hep-ph].

[27] Richard J. Hill and Mikhail P. Solon. "WIMP-nucleon scattering with heavy WIMP effective theory". *Phys. Rev. Lett.* 112 (2014), p. 211602. DOI: 10.1103/PhysRevLett.112.211602. arXiv: 1309.4092 [hep-ph].

[28] M Neubert. "Heavy-Quark Effective Theory". *Subnucl. Ser.* hep-ph/9610266. CERN-TH-96-281. 34 (1996), 98–165. 47 p.

[29] Gerhard Buchalla. "Heavy Quark Theory". hep-ph/0202092. CERN-TH-2002-018 (2002), 50 p.

[30] Karoline Kopp and Takemichi Okui. *Effective Field Theory for a Heavy Majorana Fermion*. Nov. 2011. DOI: 10.1103/PhysRevD.84.093007. arXiv: 1108.2702v2 [hep-ph].

[31] J. Gasser and H. Leutwyler. "Chiral Perturbation Theory: Expansions in the Mass of the Strange Quark". *Nucl. Phys.* B250 (1985), pp. 465–516. DOI: 10.1016/0550-3213(85)90492-4.

[32] Steven Weinberg. "Nuclear forces from chiral Lagrangians". *Phys. Lett.* B251 (1990), pp. 288–292. DOI: 10.1016/0370-2693(90)90938-3.

[33] C. C. Nishi. "Simple derivation of general Fierz-type identities". *Am.J.Phys.* 73 (Jan. 2005), pp. 1160–1163. DOI: 10.1119/1.2074087. arXiv: hep-ph/0412245v4 [hep-ph].

[34] Evgeny Epelbaum. "Nuclear Forces from Chiral Effective Field Theory: A Primer". 2010. arXiv: 1001.3229 [nucl-th].

[35] Nikhil Anand, A. Liam Fitzpatrick, and W. C. Haxton. "Weakly interacting massive particle-nucleus elastic scattering response". *Phys. Rev.* C89.6 (2014), p. 065501. DOI: 10.1103/PhysRevC.89.065501. arXiv: 1308.6288 [hep-ph].

[36] Brian Henning et al. "2, 84, 30, 993, 560, 15456, 11962, 261485, ...: Higher dimension operators in the SM EFT" (2015). arXiv: 1512.03433 [hep-ph].

[37] H. Simma. "Equations of Motion for Effective Lagrangians and Penguins in Rare B-Decays". *Z.Phys. C* 61 (July 1994), pp. 67–82. DOI: 10.1007/BF01641888. arXiv: hep-ph/9307274v1 [hep-ph].

[38] Ben Gripaios and Dave Sutherland. "An operator basis for the Standard Model with an added scalar singlet". *JHEP* 1608 (Aug. 2016), p. 103. DOI: 10.1007/JHEP08(2016)103. arXiv: 1604.07365v3 [hep-ph].

[39] C. C. Nishi. "Simple derivation of general Fierz-like identities". *Am. J. Phys.* 73 (2005), pp. 1160–1163. DOI: 10.1119/1.2074087. arXiv: hep-ph/0412245 [hep-ph].

[40] W. Buchmuller and D. Wyler. "Effective Lagrangian Analysis of New Interactions and Flavor Conservation". *Nucl. Phys.* B268 (1986), pp. 621–653. DOI: 10.1016/0550-3213(86)90262-2.

[41] B. Grzadkowski et al. "Dimension-Six Terms in the Standard Model Lagrangian". *JHEP* 10 (2010), p. 085. DOI: 10.1007/JHEP10(2010)085. arXiv: 1008.4884 [hep-ph].

[42] Landon Lehman and Adam Martin. *Hilbert Series for Constructing Lagrangians: expanding the phenomenologist's toolbox*. Tech. rep. arXiv:1503.07537. Comments: 20 pages. 2015.

[43] Brian Henning et al. "Hilbert series and operator bases with derivatives in effective field theories". *Commun. Math. Phys.* 347.2 (2016), pp. 363–388. DOI: 10.1007/s00220-015-2518-2. arXiv: 1507.07240 [hep-th].

[44] Michael A. Fedderke et al. "The Fermionic Dark Matter Higgs Portal: an effective field theory approach" (Aug. 2014), JHEP. DOI: 10.1007/JHEP08(2014)122. arXiv: 1404.2283v2 [hep-ph].

[45] Bryce S. DeWitt. "Quantum Theory of Gravity. 2. The Manifestly Covariant Theory". *Phys. Rev.* 162 (1967), pp. 1195–1239. DOI: 10.1103/PhysRev.162.1195.

[46] L. F. Abbott. "Introduction to the Background Field Method". *Acta Phys. Polon.* B13 (1982), p. 33.

[47] Gerard 't Hooft and M. J. G. Veltman. "Regularization and Renormalization of Gauge Fields". *Nucl. Phys.* B44 (1972), pp. 189–213. DOI: 10.1016/0550-3213(72)90279-9.

[48] Paulo Nogueira. "Automatic Feynman graph generation". *J. Comput. Phys.* 105 (1993), pp. 279–289. DOI: 10.1006/jcph.1993.1074.

[49] J. Kuipers et al. "FORM version 4.0". *Comput. Phys. Commun.* 184 (2013), pp. 1453–1467. DOI: 10.1016/j.cpc.2012.12.028. arXiv: 1203.6543 [cs.SC].

[50] Adam Alloul et al. "FeynRules 2.0 - A complete toolbox for tree-level phenomenology". *Comput.Phys.Commun.* 185 (May 2014), pp. 2250–2300. DOI: 10.1016/j.cpc.2014.04.012. arXiv: 1310.1921v2 [hep-ph].

[51] P Cvitanovic. *Group theory for Feynman diagrams in non-Abelian gauge theories*. Tech. rep. COO-2220-66. 1976.

[52] K G Chetyrkin, M Misiak, and M Münz. "Beta functions and anomalous dimensions up to three loops". *Nucl. Phys. B* 518.hep-ph/9711266. IFT-97-11. MPI-PHT-97-45. TTP-97-43. TUM-HEP-284. ZU-TH-97-16 (1997), 473–494. 22 p.

Eidesstattliche Versicherung

Ich versichere hiermit an Eides statt, dass ich die vorliegende Masterarbeit mit dem Titel ”Effective Theory for Dark Matter Direct Detection at Dimension Seven” selbständig und ohne unzulässige fremde Hilfe erbracht habe. Ich habe keine anderen als die angegebenen Quellen und Hilfsmittel benutzt sowie wörtliche und sinngemäße Zitate kenntlich gemacht. Die Arbeit hat in gleicher oder ähnlicher Form noch keiner Prüfungsbehörde vorgelegen.

Ort, Datum	Unterschrift

Belehrung

Wer vorsätzlich gegen eine die Täuschung über Prüfungsleistungen betreffende Regelung einer Hochschulprüfungsordnung verstößt handelt ordnungswidrig. Die Ordnungswidrigkeit kann mit einer Geldbuße von bis zu 50.000,00 € geahndet werden. Zuständige Verwaltungsbehörde für die Verfolgung und Ahndung von Ordnungswidrigkeiten ist der Kanzler/die Kanzlerin der Technischen Universität Dortmund. Im Falle eines mehrfachen oder sonstigen schwerwiegenden Täuschungsversuches kann der Prüfling zudem exmatrikuliert werden (§ 63 Abs. 5 Hochschulgesetz - HG -).

Die Abgabe einer falschen Versicherung an Eides statt wird mit Freiheitsstrafe bis zu 3 Jahren oder mit Geldstrafe bestraft.

Die Technische Universität Dortmund wird ggf. elektronische Vergleichswerkzeuge (wie z.B. die Software ”turnitin”) zur Überprüfung von Ordnungswidrigkeiten in Prüfungsverfahren nutzen.

Die oben stehende Belehrung habe ich zur Kenntnis genommen.

Ort, Datum	Unterschrift